Transit Timetable Synchronization for Transfer Time Minimization

Mojtaba Abdolmaleki, Neda Masoud, Yafeng Yin
Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109, United States,
mojtabaa@umich.edu, nmasoud@umich.edu, yafeng@umich.edu,

1. Introduction

The design process of a transit network involves multiple inter-dependent components, and is therefore complicated. As such, the transit planning problem is often divided into several subproblems including transit network design, timetable design, vehicle scheduling, crew (drivers) assignment, and fleet maintenance scheduling. This paper is concerned with the timetable design subproblem, which determines the departure times of vehicles from depot(s) to optimize an objective function, e.g., the total transfer waiting time. Transfer time contributes substantially to the disutility associated with transit use. Therefore, synchronization of trips in timetables could improve the perceived quality of transit and increase transit ridership, and also potentially alleviate congestion at bus stops by reducing dwelling times(Ibarra-Rojas and Rios-Solis 2012, Bookbinder and Desilets 1992).

2. Problem statement

The most relevant prior paper to our research is by Shafahi and Khani (2010), which proposed a scheduling model to reduce the total transfer waiting time in a transit service. The model considered fixed headways for each transit line and constant number of passengers for each pair of intersecting lines. This work represents a nice tradeoff between tractability and realism. For small and medium network sizes, it also provides a tight bound on the optimal solution. However, there are three main assumptions that make this work less applicable in practice. First, Shafahi and Khani (2010) did not consider the interaction of line headways over consecutive planning periods. This results in overestimating the effect of transit scheduling over the entire planning horizon. Second, it assumed the number of transferring passengers to be constant, which is not the case in many real-world networks. Lastly, as the size of the problem grows, they cannot provide tight bounds for their optimal solution. This makes it hard to conclude whether the reduction in the total transfer waiting

time stems from the effectiveness of timetable synchronization, or is just the result of a growing optimality gap. In this paper, we resolve these issues by generalizing the model to include several time horizons among which the line headways and the number of transferring passengers can change. Moreover, we exploit the close relationship between this problem and the problem of finding the maximum directed cut in a weighted graph (hereafter referred to as MAX DICUT) to take advantage of the specialized algorithms developed for MAX DICUT.

3. Solution Methodology

In this paper we formulate the transfer waiting time minimization problem as an optimization problem with congruence constraints, i.e., constraints that involve modular equations. While this problem is in general NP-hard, we investigate network structures for which it is possible to find the optimal solution in polynomial time. As it is hard to find a global optimum for the general case, we conduct a local search by solving a MAX DICUT problem on a graph that is defined based on the structure of the transit system. Then, we customize and apply a commonly used algorithm to approximate the MAX DICUT problem, ALG1, on a real case study, and compare the quality of our solutions with those available from the literature. Finally, we investigate the general setting in which the headways and number of transferring passengers are not constant. We propose a recursive algorithm to find the total transfer waiting time for passengers, and prove its efficiency. We improve this algorithm by adding a local search to it, ALG2, and execute it on a large-scale transit network.

4. Numerical Studies

4.1. Example 1

In this Section we implement ALG1 on a real case study used in Shafahi and Khani (2010). The reference network is the Mashhad city bus transit service, which consists of 278 lines (139 two-way lines) and 3,618 stopping stations, 841 of which are transfer stations. The minimum and maximum headways in the network are 2 and 165 minutes, respectively. The parameters used are borrowed from Shafahi and Khani (2010), except for transfer times and stopping times at transfer stations, which were not reported. We used several sets of values for transfer times and stopping times at transfer stations, and observed that the results were not sensitive to choice of these parameters. Note that it is only the difference between transfer times and stopping times at transfer stations that affect the objective value (1a). As such, here we report the results for a case where transfer times and stopping times at transfer stations are equal to each other.

Figure 1 shows that ALG1 converges to a 4.8% gap within the first few iterations (obtained in one second). The quality of the solution, even after one second, is promising. We reached a relative gap of 2.28% in 6 minutes. Based on the results reported in Shafahi and Khani (2010), our bound

on the optimal solution is much tighter than the bound they obtained using the solver (6.4%), which is obtained in 30 minutes. Furthermore, the solution accuracy of ALG1 is comparable to the accuracy obtained in Shafahi and Khani (2010), where they ran a genetic algorithm model for over two and a half hours to obtain a solution.

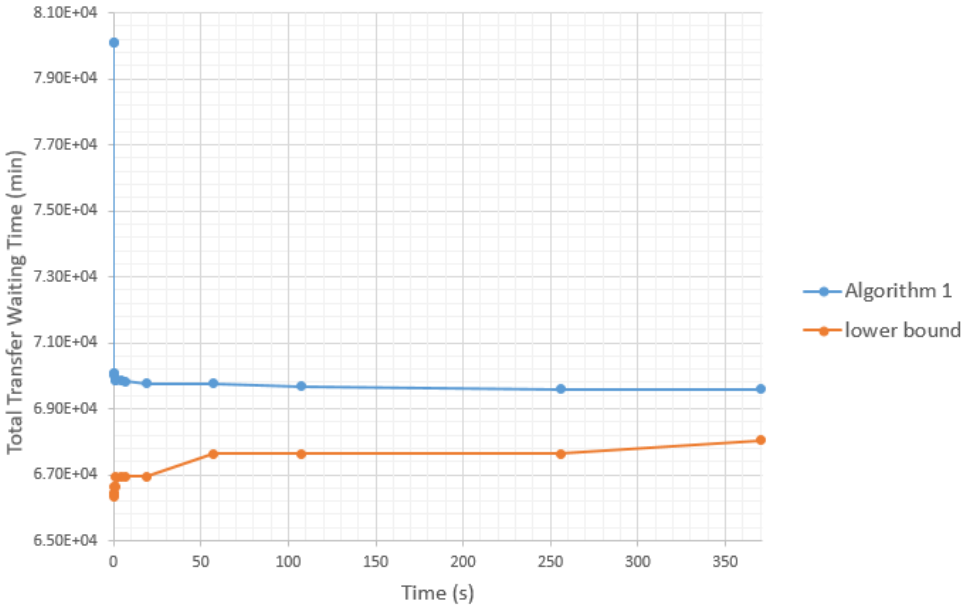


Figure 1 **Trend of objective function improvement**

4.2. Local Search For The General Case

To assess the performance of ALG2, we implement it on the case study of the Mashhad city. To do so, we generate a profile for line headways with two peak periods, one peaking in the morning rush hour and the other in the evening rush hour. The headways during the non-peak periods are set to γ times the headways for peak periods, and have been chosen proportionate to the headways in Section 4.1. We also set the number of transferring passengers in the peak period to α times the number of transferring passengers in the peak period. Figure 2 displays the rate at which the objective function improves using ALG2 for $\gamma = 2, \alpha = 1$. As this figure demonstrates, the total transfer waiting time decreases by 9.3% in less than a minute.

5. Concluding Remarks

In this paper we propose a new modeling framework with congruence constraints to minimize the total transfer time in a transit network. We show that this problem has the MAX-CUT problem as a subproblem, and therefore it is NP-hard. We investigate network structures for which it is possible to find the optimal solution in polynomial time. As it is computationally prohibitive to solve the

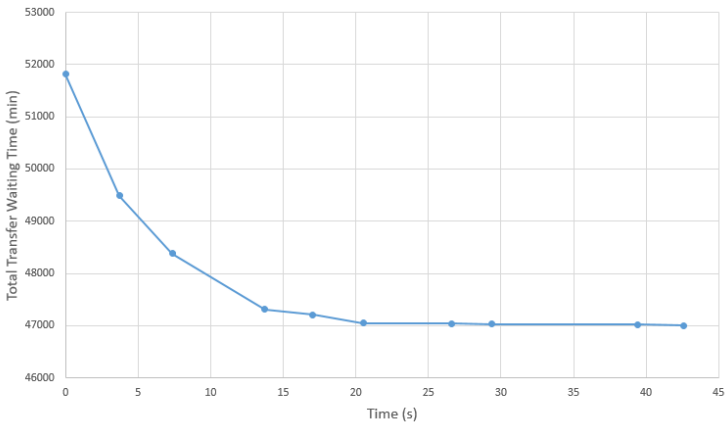


Figure 2 **Trend of objective function improvement for $\gamma = 2$ and $\alpha = 1$**

general case of the problem to optimality, we conduct a local search by solving a MAX DICUT problem. Finally, we customize and apply a common algorithm previously used to approximate the MAX DICUT problem on a real-world case study, and compare the quality of our solutions and our solution times with those presented in the literature. Lastly, we relax the assumption of the headways and the number of transferring passengers between any pair of lines being constant, and propose a local search algorithm to find a solution in this general setting. As the transit timetable synchronization problem can be used as a sub-problem in larger optimization problems that seek to optimize multiple aspects of transit operations, the proposed algorithm can help advance the state of art and practice in transit planning.

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