A Novel Method-of-Fragments Algorithm for Optimising Drayage Operations

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1. Introduction

Although representing by far the shortest leg of international container transportation, inland container transportation (or *drayage*) accounts for 80% of the total transportation cost. This disproportionate share of the costs has led to considerable interest from both industry and the research community in optimising drayage operations. Furthermore, the significant environmental and social benefits (via pollution and road congestion reduction) afforded by efficiently planned drayage operations enjoy considerable interest from a governmental perspective.

Optimising drayage operations consists of determining a least-cost transportation plan for a fleet of trucks to move a fixed set of containers over a given planning horizon. Due to the complexity of drayage operations, many authors make a number of simplifying assumptions in their optimisation models, such as trucks with unit capacity, homogeneous fleets, homogeneous container sizes, and a single source/sink location for containers, amongst others. These assumptions make the drayage problem considerably easier to solve, however have the effect of leading to models that are far removed from the realities of drayage operations in practice.

Recent work by Moghaddam et al. (2019) combats this by developing a novel flexible request model of drayage operations that models the operational complexities without the simplifying assumptions typically seen within the drayage literature. The authors subsequently develop a sophisticated mixed integer programming (MIP) formulation that performs well on small instances, however struggles to solve realistically-sized problems to optimality (Pearce and Moghaddam 2019).

The fragment-based method proposed in the recent work by Alyasiry, Forbes, and Bulmer (2019) has shown significant promise as a new state-of-the-art methodology for solving the pickup and

delivery problem with time windows (PDPTW), of which the drayage problem is a generalisation. In this work we propose a novel exact method for optimising drayage operations based on the method-of-fragments.

2. Problem Description

The key differentiator between drayage operations and a typical PDPTW is the operational requirements of the requests. The requests may be partitioned into two types: *single requests*, which must be picked up from their pickup location and delivered to their delivery location (standard to PDPs), and *composite requests*, which must be picked from their pickup location, transported to an intermediate customer location for un/loading, and subsequently transported to their delivery location. Composite requests may be further partitioned into *live composite requests* in which the transporting vehicle must stay with the container during the un/loading process, and *separable composite requests* which allow the container to be dropped at the customer location for later collection (potentially by a different vehicle).

The drayage problem seeks to maximise the number of requests that can be covered by the fleet whilst minimising the operational cost of transporting these requests. Truck routes must respect all time window and capacity constraints whilst also satisfying per truck shift-length constraints and ensuring that an un/loading service time is maintained at customer locations for composite requests. In the case of separable composite requests, this last requirement means that pairs of routes may be incompatible due to the minimum separation of the first and second halves of a separable request (due to the implied precedence constraint).

3. A Method-of-Fragments Approach

The method-of-fragments technique introduced by Alyasiry, Forbes, and Bulmer (2019) is based on the concept of a fragment—a sequence of pickup and delivery requests starting and ending with an empty vehicle that is not empty at any intermediate point. As fragments must respect all load, coupling, and timing constraints, fragments may then be 'chained together' with empty-repositioning arcs via a time-discretised network flow problem with side constraints to construct vehicle routes. In constructing this time-discretised network, the arrival times of fragments and repositioning arcs are rounded down. This can lead to infeasible routes due to time-window violations and the existence of subtours; these are cut off by lazy constraints (which are in turn equivalent to combinatorial Benders cuts on an MTZ-style feasibility subproblem). The method has been shown to be highly effective where all fragments may be generated a priori.

The application of the method-of-fragments to the drayage problem requires significant extension of the methodology to deal with the intermediate customer locations, shift-length constraints, the

inter-truck precedence requirements implied by separable composite requests, and high degree of symmetry occasioned by frequent co-location of pickups and deliveries. The key novelties and methodological advancements of this work are:

1. Development of considerable theory surrounding fragment dominance rules for requests with precedence constraints, and dominance rules that may validly be applied for incomplete fragments (during the generation process).

2. A new highly-parallelisable stage-wise fragment generation algorithm that incorporates dominance checking of incomplete fragments during generation.

3. Two network flow models for the relaxed network: a vehicle-based formulation modelling individual vehicle routes (allowing for feasibility cuts with high specificity) and a class-based formulation that pools together vehicles of the same type (dramatically reducing symmetry at the cost of cut specificity). As previous work on fragments has not considered heterogeneous fleets, this work represents the first empirical comparison of the efficacy of the two approaches.

4. Development of a new feasibility subproblem that identifies the two new sources of timing infeasibility: per truck shift-length violations and precedence violations for separable requests, in addition to the formulation of new cuts to carefully handle these new sources of infeasibility.

4. Computational results

The MIP formulation developed by Moghaddam et al. (under second round of revision 2020) and the two method-of-fragments implementations were run on the instances from Moghaddam et al. (2019). These instances are based closely on real-world operational data from a logistics company in Brisbane, Australia. Each instance has between 43 and 89 requests, time windows from 30 minutes to 3 hours, and a heterogeneous fleet of trucks of up to 3 TEU capacity. All code was written in Python 3.6 and used the callable library of Gurobi 8.1 to solve the MIPs with 8 threads.

Table 1 displays the run times (in seconds, limited to one hour) and final percentage gap of each of the algorithms across a representative set of instances. Note that percentage gaps are reported only if the algorithm did not solve the instance to optimality within the time limit.

The table shows that whilst the vehicle-based fragments formulation solves eight additional instances to optimality as compared to the MIP method of Moghaddam et al. (2019), it struggles to obtain non-trivial solutions for some of the larger problems. The class-based fragments formulation however solves all but one of the instances to optimality within the time limit (with this instance reaching a gap of just 0.04%). Additionally, across instances solved to optimality the class-based fragments method arrives at these optimal solutions 95.3% faster than the MIP method and 92.8% faster than the vehicle-based fragments method: demonstrating the superiority of the class-based fragments method over the previous state-of-the-art (Moghaddam et al. 2019).

Table 1 Computational results on real-world resampled instances

	MIP		Vehicle-based		Class-based	
Instance	**Gap** (%)	**Time** (s)	**Gap** (%)	**Time** (s)	**Gap** (%)	**Time** (s)
15-LCF-35		0.52		0.25		**0.04**
19-LCF-25		218.54		36.69		**1.72**
23-LCF-15	0.30	3600.00	0.01	3600.00		**3.84**
45-LCF-80		1.07		0.43		**0.06**
43-LCF-70	1.85	3600.00		22.01		**1.03**
41-LCF-60	1.25	3600.00	0.07	3600.00		**9.16**
55-LCF-125	2.76	3600.00		1.96		**0.24**
53-LCF-115	10.06	3600.00		619.25		**21.13**
51-LCF-105	2.66	3600.00		1269.98		**25.40**
65-LCF-170	0.40	3600.00		3.72		**0.33**
63-LCF-160	1.95	3600.00		1508.46		**5.81**
61-LCF-150	68.55	3600.00	*	`m`		**1117.78**
32-BCF-50	0.11	3600.00		122.53		**5.09**
31-BCF-45	2.09	3600.00	*	3600.00		**1178.06**
30-BCF-40	21.59	3600.00	*	`m`	0.04	**3600.00**
48-BCF-95	0.95	3600.00		353.90		**33.00**
47-BCF-90	67.47	3600.00	*	3600.00		**888.37**
#Optimal	3/17		11/17		16/17	

'MIP' reports the computational results of the MIP method developed by Moghaddam et al. (2019), 'Vehicle-based' the method-of-fragments implementation with separate variables for each truck, and 'Class-based' the method-of-fragments implementation with variables for each vehicle class. * Indicates that only the trivial solution was found within the time limit, and `m` that the algorithm terminated after exhausting the available memory.

In addition to mathematical details of the work, the conference presentation also will report on limitations of the method and examine various algorithmic enhancements and extensions.

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