Modeling Two-stage Competition on Capacity and Pricing for Transit and Parking Planning

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Transit (also known as public transport) is often advocated as a good alternative to private automobiles for satisfying the transportation needs of urban commuters. Transit systems are characterized as attractive solutions to alleviate the current environmental issues without constraining peoples mobility (Schiller, Bruun, and Kenworthy 2010, Miller 2014). The expectations from transit systems to be accessible, affordable and safe often led to a high degree of government regulation of transit systems in many parts of the world. However, these regulations often caused severe financial problems for these systems, primarily because the costs of providing such high level of service were not fully recovered by the fare revenues collected by the transit operators (Vuchic 2005). This situation raises the question of what would happen if a transit system were to place more emphasis on the profitability (or loss reduction) objective.

In this paper, we introduce a versatile mathematical modeling and algorithmic framework that allows transit operators to explore the impacts of incorporating the profit maximization objective into their capacity planning and pricing decisions. Given that driving (and subsequent parking) of personal vehicles is a major alternative to using transit in many cities, especially for daily commuting needs, our approach consists of a game-theoretic model of the interactions between the decisions of transit and parking operators. Under this approach, transit operator and parking operator are both modeled as decision-makers whose optimal decisions depend on each others decisions.

An integrated approach to transit and parking planning was considered by Cavadas and Antunes (2018). They developed an optimization approach to minimize the joint operating deficit of the transit and parking operators while ensuring required minimum levels of mobility in a city. However, and to the best of our knowledge, the topic of transit and parking interactions under competition has not been explored in any part of the existing literature.

We propose a two-stage game-theoretic framework where both transit and parking operators optimize their capacity decisions in the first stage (denoted as Stage I) and their pricing decisions in the second stage (denoted as Stage II). Stage I deals with decisions regarding the level of supply by each operator: for parking operators, these include parking capacities offered in each paid parking lot, while for transit operators, these include the frequencies for each transit route and the fleet size adjustments. Stage II focuses on the short-term decisions regarding the prices: for parking operators, it is the parking fee, while for transit operators, it is the transit fare.

In the Stage II game, the operators optimize their pricing strategies to maximize their revenue since the capacity decisions are predetermined. The passenger choice is described by a logit model of the generalized travel costs. The generalized cost for traveling with transit is the sum of in-vehicle time cost, transit fare, the access cost, and the discomfort cost. The discomfort cost is an increasing nonlinear function of the number of passengers using transit and a decreasing function of the transit frequency. The cost for traveling with car is the sum of cost for travel time, cost for total distance traveled, parking fee, and cost of cruising for parking in the destination zone. Here, the term cruising refers to the process of drivers driving around on roads in certain parts of the city or driving through a large parking garage, in search of vacant parking spots.

In the Stage I game, the transit and parking operators optimize their capacity decisions to maximize their profits, as shown in Equation (1) and (2). The profit for transit operators is consisted of revenue ($\overline{OF}^{SS}_{Transit}\left([f_k]_{k\in\hat{T}}, [S^{Car}_j]_{j\in\mathbf{Z}}\right)$), which is a function of transit frequencies and parking capacities, operating cost ($E^{\cdot}_{Transit}\sum_{k\in\hat{T}}(f_K \cdot D^K_{kTransit})$) and ownership cost ($E^F_{Transit}\cdot\sum_{k\in\hat{T}} f_k$) of vehicles, and expense/revenue from buying or selling transit vehicles ($-M^{PV}_B\cdot v^{PV} + M^{SV}_B\cdot v^{SV}$). The profit for parking operators is consisted of revenue ($\overline{OF}^{SS}_{Car}\left([f_k]_{k\in\hat{T}}, [S^{Car}_j]_{j\in Z}\right)$), operating cost in terms of number of slots ($M^V_{Car}\cdot\sum_{j\in Z} S^{Car}_j\cdot\gamma_j$), and parking lot expansion cost ($E^F_{Car}\cdot\sum_{j\in\mathbf{Z}} y^P_j$).

$$\begin{aligned} OF^{FS}_{Transit} =& \overline{OF}^{SS}_{Transit}\left([f_k]_{k\in\hat{T}}, [S^{Car}_j]_{j\in\mathbf{Z}}\right) - E^{\cdot}_{Transit}\sum_{k\in\hat{T}}\left(f_K\cdot D^K_{kTransit}\right) - E^F_{Transit}\cdot\sum_{k\in\hat{T}} f_k \\ & - M^{PV}_B\cdot v^{PV} + M^{SV}_B\cdot v^{SV} \end{aligned} \tag{1}$$

$$OF^{FS}_{Car} = \overline{OF}^{SS}_{Car}\left([f_k]_{k\in\hat{T}}, [S^{Car}_j]_{j\in Z}\right) - M^V_{Car}\cdot\sum_{j\in Z} S^{Car}_j\cdot\gamma_j - E^F_{Car}\cdot\sum_{j\in\mathbf{Z}} y^P_j \tag{2}$$

As acknowledged by Colson, Marcotte, and Savard (2007), bi-level optimization problems are challenging to solve due to their non-convex and NP-hard nature. We proposed an approximate solution method consists of the following three steps: proposed approximate solution method consists of the following three steps:

1. Find pure strategy Nash equilibrium (PSNE) of Stage II for a selected set of combinations of the decisions made during Stage I by both operators;

2. Approximate the revenues calculated for Stage II during Step 1 above for each operator as functions of the decisions made by both operators during Stage I using quadratic functions;

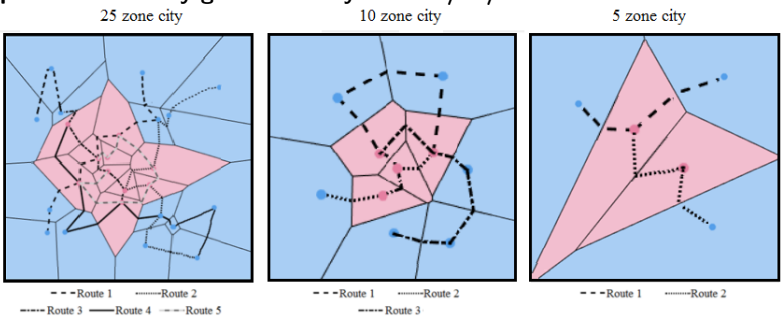
3. Use these revenue approximations to find PSNE of Stage I.

Thus, Step 1 and Step 3 require repeatedly solving combinatorial, non-linear and non-convex optimization formulations. Step 2 involves approximating the revenues corresponding to the Stage II PSNE as functions of Stage I decisions. We used polynomial approximations which provide a good fit to the exact revenue values calculated at the Stage II PSNEs, and capture the gross properties of the relationship between the capacities and frequencies of Stage I and the revenues at the Stage II PSNE. In order to reduce the long run time caused by the nonlinearity in Stage II game, a heuristic method is presented to reduce the run time by more than 90% in all numerical cases.

By investigating into extensive computational experiments, we explored the existence and uniqueness of Nash equilibrium. Consistently promising and comprehensive computational results strongly suggest that the existence and uniqueness properties of the Stage II PSNE are likely to hold in practical instances of our two-stage model. It justifies the adequacy of PSNE as the solution concept for the Stage II game.

In order to test our algorithm on generated cities with different number of zones and populations as shown in Figure 1. Since the nonlinearity has made solving Stage II optimization very time-consuming.

Figure 1 **An example of a randomly generated city with 25/10/5 zones.**



After applying our model to different scales of cities, we found that we are able to increase the profitability significantly for both parking and transit agencies. For small cities, the parking

operators profit increases to three times the original value, but there is still a loss made by the transit operator, though the value has decreased by 82% compared to the prevalent loss value. In middle-size cities, the transit and parking operators both significantly improve their profits, and in particular, the transit operator becomes profitable and posts strong profits (increased by 477%).

We also found that the decisions of the two operators are shown to be highly interdependent, and changes in parameters of one operator can sometimes affect equilibrium decisions of both operators. For example, when the vehicle selling price is lower for transit operators, the parking operators will also respond to that change by adjusting it's own parking capacities in different area (in our case, it's increase 7 parking spots in zone 1 and reduce 7 parking spots in zone 5).

Our paper made three major contributions: 1) Ours is the first study to model the competition between transit and parking operators using a two-stage game-theoretic model. 2) We develop a new solution method which uses a semi-approximate approach to make the two-stage model tractable and solvable. 3) We analyze and demonstrate the existence and uniqueness properties of PSNEs for both stages of our game through detailed computational experiments. This analysis validates our modeling choices and our choice of the game-theoretic solution concept. 4) Finally, we develop an original end-to-end computational framework to simulate, solve and evaluate two-stage games between the transit and parking operators for a series of case study networks.

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