# Pessimistic Evasive Flow Capturing Problems

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## 1. Introduction

The evasive flow capturing problem (EFCP) is to locate a set of law enforcement facilities to intercept unlawful flows. One application of the EFCP is the location problem of weight-in-motion systems deployed by authorities to detect overloaded vehicles characterized by evasive behavior. The EFCP is typically studied as a leader-follower game using the corresponding bilevel optimization formulations. The government is a leader determining the location of law enforcement facilities, and travelers are followers selecting their choices of paths based on the decision of the government, which is typically assumed to be the shortest unintercepted path. Further, when there are multiple optimal solutions for followers in the lower-level, friendly or optimistic behavior of followers are assumed in the literature (Arslan, Jabali, and Laporte 2018). Such assumptions on the optimizing behavior of travelers may be appropriate in certain circumstances; however, it does not represent the most general case of the EFCP. Moreover, such an optimistic approach may lead to less desirable solutions, especially when followers are not optimizing decision-makers.

In this study, we present the pessimistic formulation of the EFCP considering two different perspectives. First, we relax the optimizing behavior by letting travelers select any unintercepted path within a deviation tolerance. This approach is to consider boundedly rational or satisficing travelers, who select any path whose distance is within a certain threshold of the shortest path. In fact, empirical studies focused on drivers choice of routes and network behavior show limitations of the assumptions regarding the rationality of drivers (Nakayama, Kitamura, and Fujii 2001). Second, while the exact path choice of followers among many possible choices remains ambiguous, we consider the worst-case to the leader and formulate pessimistic bilevel optimization problems. Using Albany network, we show that the pessimistic solutions may prevent damage to the network 14% more compared to optimistic solutions.

The contributions of this study can be summarized as follows: i) we present the generic form of the EFCP considering bounded rationality of travelers; ii) we propose the pessimistic formulation, which yield a robust network design and allow control over the level of pessimism on the behaviors of drivers; iii) we propose exact cutting plane algorithms to solve the resulting pessimistic formulation.

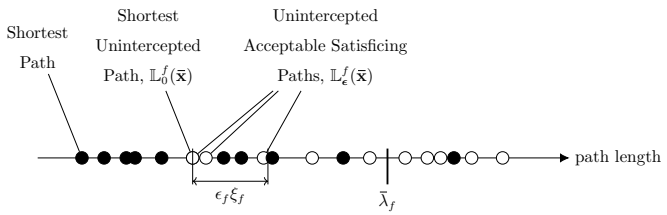



**Figure 1** **The path choice of followers is ambiguous among unintercepted acceptable satisficing paths in Assumption.**

## 2. Pessimistic Formulation

In a given network $\mathcal{G}(\mathcal{N}, \mathcal{A})$, with a set of nodes $\mathcal{N}$ and a set of arcs $\mathcal{A}$, let us consider a set of vehicle flows, $\mathcal{F}$. Each flow $f \in \mathcal{F}$ is characterized by an origin-destination pair and a deviation tolerance factor $\lambda_f > 0$. Let $\xi_{s_f t_f}$ represent the shortest path between origin node $s_f$ and destination node $t_f$ for flow $f$. A path is called *acceptable* for flow $f$, if its length is at most $\bar{\lambda}_f$, where $\bar{\lambda}_f = \lambda_f \xi_{s_f t_f}$. A path is called *unintercepted* for flow $f$, if it enables traversing without passing any law enforcement facilities. Then for each flow $f \in \mathcal{F}$, $\bar{\lambda}_f$ induces a restricted set of nodes $\mathcal{N}_f \subset \mathcal{N}$ and a restricted set of arcs $\mathcal{A}_f \in \mathcal{A}$. For each arc $(i,j) \in \mathcal{A}_f$ we have $\xi_{s_f i} + d_{ij} + \xi_{j e_f} \leq \bar{\lambda}_f$. Then $\mathcal{N}_f$ represents all nodes in $\mathcal{A}_f$. The EFCP naturally fits the bilevel setting, where the government (the leader) decides the location of law enforcement facilities and unlawful travelers (followers) aim to drive on unintercepted acceptable paths. Then the set of leader's feasible decisions can be defined as follows:

$$\mathbf{X} = \big\{\mathbf{x} : x_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}\big\}. \tag{1}$$

where $x_{ij}$ is a binary variable, whose value equals 1 if a law enforcement facility is located on arc $(i,j)$ and 0 otherwise. Accordingly, vector $\mathbf{x}$ represents all $x_{ij}$ variables for all $(i,j) \in \mathcal{A}$. We define the set of feasible reactions of followers for any given leader's decision $\mathbf{x} \in \mathbf{X}$:

$$\mathbf{L}^f(\mathbf{x}) = \left\{ (\mathbf{r}^f, u_f) \;\middle|\; \begin{array}{ll} \displaystyle\sum_{(i,j)\in\mathcal{A}_f} r_{ij}^f - \sum_{(j,i)\in\mathcal{A}_f} r_{ji}^f = \begin{cases} u_f & \text{if } i = s_f \\ -u_f & \text{if } i = t_f \\ 0 & \text{otherwise} \end{cases} & \forall i \in \mathcal{N}_f, \\ r_{ij}^f \leq 1 - x_{ij} & \forall (i,j) \in \mathcal{A}_f, \\ u_f, r_{ij}^f \in \{0,1\} & \forall (i,j) \in \mathcal{A}_f \end{array} \right\} \tag{2}$$

where $u_f$ is a binary variable whose value is equal to 1 if flow $f$ is unintercepted and 0 otherwise. Another binary variable $r_{ij}^f$ indicates if arc $(i,j)$ is selected for the follower's path or not. Then vector $\mathbf{r}^f$ represents all $r_{ij}^f$ variables for all $(i,j) \in \mathcal{A}_f$. We use $\mathbf{L}(\mathbf{x})$ to denote the Cartesian product $\prod_{f\in\mathcal{F}} \mathbf{L}^f(\mathbf{x}) = \mathbf{L}^1(\mathbf{x}) \times \mathbf{L}^2(\mathbf{x}) \times \cdots \times \mathbf{L}^{|\mathcal{F}|}(\mathbf{x})$.

To model the behavior of followers, we utilize the bounded rationality of travelers as illustrated in Figure 1. While there are five unintercepted acceptable paths available, we assume that followers

will only choose the first three paths, since the lengths of those three paths are 'short enough' within a certain threshold. The first three paths are called *satisficing* paths, which are formally defined as follows: for each follower $f \in \mathcal{F}$, a path is called *satisficing* with threshold $\epsilon_f > 0$, if its length is shorter than $(1+\epsilon_f)\xi_f$, where $\xi_f$ is the length of the shortest unintercepted path. Some satisficing paths are also *subpath*-satisficing, where if any subpath, including itself, is a satisficing path with the same threshold $\epsilon_f$. We assume the following behavior of the followers (unlawful travelers):i)the followers drive on an unintercepted acceptable path, but the exact path choice is ambiguous among subpath-satisficing paths; ii)If no unintercepted acceptable path is available, then the follower does not travel. To provide a formulation that is consistent with the above assumptions, we introduce new variables and sets. Let $\bar{\boldsymbol{\lambda}} = (\bar{\lambda}_f : f \in \mathcal{F})$ and $\boldsymbol{\epsilon} = (\epsilon_f : f \in \mathcal{F})$. Note that $\delta$ is a small positive constant such that $\delta < d_{ij}$ for all $(i,j) \in \mathcal{A}$. For any $\boldsymbol{\epsilon} \geq \mathbf{0}$, we define

$$\mathfrak{L}^f_{\boldsymbol{\epsilon}}(\mathbf{x}; -\boldsymbol{\sigma}^f) = \underset{(\mathbf{r}^f, u_f) \in \mathbf{L}^f(\mathbf{x})}{\arg\max} \left\{ \frac{\bar{\lambda}_f + \delta}{1+\epsilon_f} u_f - \sum_{(i,j)\in\mathcal{A}_f} (d_{ij} - \sigma^f_{ij}) r^f_{ij} \right\}. \tag{3}$$

$$\boldsymbol{\Sigma}^f_{\boldsymbol{\epsilon}} = \left\{ \boldsymbol{\sigma}^f : 0 \leq \sigma^f_{ij} \leq \frac{\epsilon_f}{1+\epsilon_f} d_{ij} \quad \forall (i,j) \in \mathcal{A}_f \right\}$$

$$\mathbb{L}^f_{\boldsymbol{\epsilon}}(\mathbf{x}) = \left\{ (\mathbf{r}^f, u_f) \in \mathfrak{L}^f_{\boldsymbol{\epsilon}}(\mathbf{x}; -\boldsymbol{\sigma}^f) : \boldsymbol{\sigma}^f \in \boldsymbol{\Sigma}^f_{\boldsymbol{\epsilon}} \right\} \tag{4}$$

where $\mathbb{L}_{\boldsymbol{\epsilon}}(\mathbf{x}) = \prod_{f\in\mathcal{F}} \mathbb{L}^f_{\boldsymbol{\epsilon}}(\mathbf{x})$, $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} = \prod_{f\in\mathcal{F}} \boldsymbol{\Sigma}^f_{\boldsymbol{\epsilon}}$. We present a bilevel pessimistic formulation as follows:

$$\begin{aligned} \mathbf{PeBM}_{\boldsymbol{\epsilon}}: \quad & \underset{\mathbf{x}}{\text{minimize}} && \left[ \sum_{(i,j)\in\mathcal{A}} w_{ij} x_{ij} + \max_{(\mathbf{r},\mathbf{u})\in\mathbb{L}_{\boldsymbol{\epsilon}}(\mathbf{x})} \sum_{f\in\mathcal{F}} \sum_{(i,j)\in\mathcal{A}_f} h^f_{ij} r^f_{ij} \right] \\ & \text{subject to} && x_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A}. \end{aligned}$$

## 3. Solution Methods

The pessimistic formulation results in trilevel optimization problems, where the master problem based on the decision of the leader is followed by the worst-case problem (WCP). The latter problem itself has a bilevel structure. We present the cutting plane algorithm for $\mathbf{PeBM}_{\epsilon}$ as follows:

$$\mathbf{Master}: \quad \underset{\mathbf{x},\mathbf{r},\mathbf{u}}{\text{minimize}} \quad \left[ \sum_{(i,j)\in\mathcal{A}} w_{ij} x_{ij} + \sum_{f\in\mathcal{F}} \sum_{(i,j)\in\mathcal{A}_f} h^f_{ij} r^f_{ij} \right] \tag{5}$$

$$\text{subject to} \quad \mathbf{x} \in \mathbf{X} \tag{6}$$

$$(\mathbf{r},\mathbf{u}) \in \mathbf{L}(\mathbf{x}) \tag{7}$$

$$\{\text{cuts are added iteratively}\}.$$

$$\mathbf{WCP}_{\boldsymbol{\epsilon}}(\bar{\mathbf{x}}): \quad \underset{\mathbf{r},\mathbf{u}}{\text{maximize}} \quad \sum_{f\in\mathcal{F}} \sum_{(i,j)\in\mathcal{A}_f} h^f_{ij} r^f_{ij} \tag{8}$$

$$\text{subject to} \quad (\mathbf{r},\mathbf{u}) \in \mathbb{L}_{\boldsymbol{\epsilon}}(\bar{\mathbf{x}}) \tag{9}$$




(a) Albany network (b) Random network

**Figure 2 The value of the pessimistic formulation**

For each $f \in \mathcal{F}$, we compare the paths chosen by $\bar{\mathbf{r}}^f$ for master problem and $\widehat{\mathbf{r}}^f$ for WCP. We focus on the distinct subpaths $\bar{p}$ and $\widehat{p}$ and add the following cuts for each $f \in \mathcal{F}$:

- If $\bar{u}_f$=1 and $\widehat{u}_f$=1, we add the following cut: $\sum_{(i,j)\in\widehat{p}} x_{ij} \geq 1 - |\bar{p}| + \sum_{(i,j)\in\bar{p}} r^f_{ij}$.
- If $\bar{u}_f$=0 and $\widehat{u}_f$=1, we add the following cut: $\sum_{(i,j)\in\widehat{p}} x_{ij} \geq 1 - u_f$.

Cut 1 indicates that at least one arc in $\widehat{p}$ must be intercepted to reroute flow $f$ to $\bar{p}$. Cut 2 indicates that at least one arc in subpath $\widehat{p}$ must be intercepted to disable traversing of flow $f$.

## 4. Results

We use real Albany network and randomly generated networks for our experiments. To illustrate the value of the proposed pessimistic formulation we measure the difference in prevented total damage under pessimistic and optimistic solutions and denote it as VPS. As shown in Figure 2a, the value of VPS increases as we increase the value of $\epsilon$. Indeed, the value of VPS can reach up to 14% highlighting the importance of the pessimistic formulation. Similarly, for randomly generated networks VPS value can reach up to 24% depending on cost of installation $w_{ij}$ on arc $(i, j)$ as shown in Figure 2b.

## 5. Conclusion

We present and solve the pessimistic formulations of the Evasive Flow Capturing Problem (EFCP). We look at satisfying travelers, who select any path whose distance is within a certain threshold of the shortest path and let the followers select the most damaging paths. The damage caused to a network by unlawful travelers under optimistic solutions may be up to 14% higher compared to pessimistic solutions on the Albany network. While installations costs are typically high under the pessimistic solutions, they result in a long term benefit in preserving the network infrastructure.

## References

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