Distributed Approaches for Traffic Signal Control using Alternating Direction Method of Multipliers

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1. Problem Statement

Recent years have witnessed a steady growth of vehicles leading to increased traffic congestion. One of the most effective ways of reducing traffic congestion is to better control traffic signals via three perspectives: 1. *cycle length*: determining the time required for a complete sequence of signal indications; 2. *split plan*: allocating the time to different phases during a signal cycle; 3. *offset*: coordinating phases of adjacent intersections to reduce vehicle stops. However, it is challenging to solve a centralized model due to the large-scale nature and complexity of the problem. We investigate decentralized and distributed approaches, such as Alternating Direction Method of Multipliers (ADMM) (see Boyd et al. (2011)), which decompose the whole problem into several subproblems while each subproblem makes decisions for one intersection in parallel after communicating with neighboring intersections.

2. Motivation

Mitigating traffic congestion is crucial for addressing environmental concerns and improving urban mobility. Timotheou, Panayiotou, and Polycarpou (2014) modeled a mixed-integer linear program (MILP) using a cell transmission model (CTM) (see Daganzo (1995)), tailored the ADMM approach to solve a relaxed linear program, and designed two distributed rounding schemes to obtain close-to-optimal solutions. However, the CTM they considered did not incorporate turns. Besides, according to their experimental results, one of the rounding scheme (DCDRR) achieved 8% optimality gap, while the other (DDVR) obtained 76.5% optimality gap with respect to the centralized MILP. In our paper, we extend the CTM to include turns and also consider a link transmission model (LTM)

(see Yperman (2007)) for computational tractability. We compare the results of applying ADMM to the linear-programming relaxation and to the original formulation of CTM- or LTM-based MILPs.

3. Approaches

We introduce the main constraints for cell/link transmission models with turns. Then we review two variants of ADMM method on solving MILPs.

3.1. CTM Constraints

We have sets $r \in \mathcal{R}$ for all intersections, and $c \in \mathcal{C} = \mathcal{E} \cup \mathcal{O} \cup \mathcal{D} \cup \mathcal{I} \cup \mathcal{V} \cup \mathcal{M}$ for ordinary/origins/destinations/intersections/diverge/merge cells. For each intersection $i \in \mathcal{R}$, we use $I_{i,j}$ to represent the intersection cells that receive a green light at phase j. We use $d(c),\ p(c)$ to represent the successor/predecessor cells of c. Parameter $D_{c,t}$ denotes the demand of origin cell c at time t; $Q_{c,t}$ and $N_{c,t}$ denote the maximum number of vehicles that can flow through/reside into cell c at time t; $W_{c,t}$ denotes the ratio between the shock-wave propagation speed and the free-flow speed; and $\beta_{l,c},\ \beta_{t,c},\ \beta_{r,c}$ are the ratio of turning left, going through, and turning right at cell c. We use $\bar{y}_{c,t},\ y_{c,t},\ n_{c,t}$ to represent the number of vehicles that are entering/leaving/inside cell c, at time t. **Binary** variables $w_{ijt} \in \{0,1\},\ i \in \mathcal{I},\ j \in \{1,2,3,4\}$ represent the signal state of phase j at intersection i at time t. We use the following MILP to capture the most important constraints in CTM:

$$
\begin{aligned}
\max_{\bar{y}_{c,t}, y_{c,t}, n_{c,t}, w_{ijt}} \quad & \sum_{c \in \mathcal{C}, t \in \mathcal{T}} y_{c,t} \\
\text{s.t.} \quad & y_{c,t} \le n_{c,t},\ \forall c \in \mathcal{C} \\
& 0 \le y_{c,t} \le q_{c,t},\ \forall c \in \mathcal{C} \\
& y_{c,t} \le W_{c,t} \bar{n}_{c+1,t},\ \forall c \in \mathcal{C}/\mathcal{V} \\
& \beta_{l,c} y_{c,t} \le W_{c+1,t}(N_{c+1,t} - n_{c+1,t}),\ \forall c \in \mathcal{V} \\
& \beta_{t,c} y_{c,t} \le W_{c+2,t}(N_{c+2,t} - n_{c+2,t}),\ \forall c \in \mathcal{V} \\
& \beta_{r,c} y_{c,t} \le W_{c+3,t}(N_{c+3,t} - n_{c+3,t}),\ \forall c \in \mathcal{V} \\
& n_{c,t+1} = n_{c,t} + \bar{y}_{c,t} - y_{c,t},\ \forall c \in \mathcal{C}
\end{aligned}
$$

where

$$
q_{c,t} = \begin{cases} Q_{c,t},\ \forall c \in \mathcal{E} \cup \mathcal{O} \cup \mathcal{M} \\ \min\{Q_{c,t}, Q_{c+1,t}/\beta_{l,c}, Q_{c+2,t}/\beta_{t,c}, Q_{c+3,t}/\beta_{r,c}\},\ \forall c \in \mathcal{V} \\ w_{ijt} Q_{c,t},\ \forall i \in \mathcal{R},\ c \in \mathcal{I}_{i,j} \\ \infty,\ \forall c \in \mathcal{D} \end{cases}
$$

$$
\bar{n}_{c+1,t} = \begin{cases} N_{c,t} - n_{c+1,t},\ \forall c \in \mathcal{E} \cup \mathcal{O} \cup \mathcal{M} \\ N_{c,t} - n_{d(c),t},\ \forall c \in \mathcal{I} \\ \infty,\ \forall c \in \mathcal{D} \end{cases}
$$

$$\bar{y}_{c,t} = \begin{cases} y_{c-1,t},\ \forall c \in \mathcal{C}/\mathcal{O}/\mathcal{M}/\mathcal{I} \\ D_{c,t},\ \forall c \in \mathcal{O} \\ \sum_{p \in p(c)} y_{p,t},\ \forall c \in \mathcal{M} \\ \beta_{c-p(c),p(c)} y_{p(c),t},\ \forall c \in \mathcal{I} \end{cases}$$

The last constraint ensures flow conservation at cell c, while the rest constraints indicate that the number of vehicle leaving cell c is limited either by the number of vehicles in the cell, by the capacity of the cell for outflow vehicles, by the capacity of the successor cell for inflow vehicles, or by the space left in the successor cell when a queue is forming.

3.2. LTM Illustration

In the CTM model, two successive cells are used to represent a given length of the road, and we have approximately 90 cells in each intersection, making the subproblem extremely large and difficult to solve. We also consider an LTM, which models the road between two adjacent intersections as a link. Thus, we obtain a fixed and relatively smaller subproblem size (about 10 links) for each intersection. To save the space, we omit the detailed model here, which remains an MILP.

3.3. ADMM on MILP

Heuristic 1 Consider a constrained optimization problem $\min\{f(x) : x \in \mathcal{S}\}$ with f convex but $\mathcal{S}$ nonconvex. We can equivalently formulate it as $\min\{f(x) : x = z, z \in \mathcal{S}\}$ and relax $x = z$ to get an augmented Lagrangian function $L(x,u) = f(x) + (\rho/2)||x - z + u||_2^2$, where u is the scaled dual variable. According to Boyd et al. (2011), ADMM has the form

$$\begin{aligned} x^{k+1} &:= \arg\min_x \left(f(x) + (\rho/2)||x - z^k + u^k||_2^2\right) \\ z^{k+1} &:= \Pi_{\mathcal{S}}\left(x^{k+1} + u^k\right) \\ u^{k+1} &:= u^k + x^{k+1} - z^{k+1} \end{aligned}$$

where $\Pi_{\mathcal{S}}$ is the projection onto $\mathcal{S}$. When $\mathcal{S} = \{x | x_i \in \{0,1\}\}$, then $\Pi_{\mathcal{S}}$ simply rounds each variable to 0 or 1, whichever is closer.

Heuristic 2 Consider a constrained optimization problem $\min\{f(x) : x \in \mathcal{C},\ x \in \{0,1\}^n\}$. According to Wu and Ghanem (2018), we can recast the problem as $\min\{f(x) : x \in \mathcal{C},\ x = y_1,\ x = y_2,\ y_1 \in \mathcal{S}_b,\ y_2 \in \mathcal{S}_p\}$, where $\mathcal{S}_b = \{y : 0 \le y \le 1\},\ \mathcal{S}_p = \{y : ||y - \frac{1}{2}\mathbf{1}||_p^p = \frac{n}{2^p}\}$. Then we can apply the traditional ADMM method with some calculations of projection onto $\mathcal{S}_b$ and $\mathcal{S}_p$.

4. Results

We test our algorithms based on Plymouth road data in Ann Arbor, Michigan, which contains 6 intersections in a corridor and 528 cells in total. All numerical experiments are conducted in Python 3.7 on a Windows 2012 Server with 128 GB RAM and an Intel 2.2 GHz processor. We

first compare the optimal objective values of the centralized MILP and the relaxed LP solved by Gurobi in Table 1 and record the objective gaps found by Gurobi when solving MILP (in the line "MILP"), and the gaps between MILP and LP (in the line "LP"). Next, we perform two heuristics

Table 1 MILP v.s. LP in Gurobi using CTM

	$N=6,\ T=8$		$N=6,\ T=20$		$N=6,\ T=50$		$N=6,\ T=100$	
	Time (s)	Gap	Time (s)	Gap	Time (s)	Gap	Time (s)	Gap
MILP	6765.57	0.01%	-	1.42%	-	4.21%	-	54.38%
LP	0.36	−2.18%	1.28	−2.77%	6.49	−4.82%	19.91	−54.67%

to directly solve the MILP using variants of ADMM, shown in Table 2. We record the optimal objective gaps with the centralized MILP and the primal residual after 1000 iterations, indicated in columns "Gap" and "Res".

Table 2 ADMM-MILP using CTM

	$N=6,\ T=50$				
	Iterations	Time	Time per iteration	Gap	Res
Heuristic 1	1000	40670.00	40.25	3.7%	0.570
Heuristic 2	1000	49118.64	52.33	3.6%	0.499

5. Conclusions

From the above two tables, we observe that the centralized LP is a very loose relaxation of the original MILP when we increase the network size (N) and optimization time (T), which could explain the big optimality gap after rounding reported in Timotheou, Panayiotou, and Polycarpou (2014). However, by leveraging ADMM directly on MILP, we obtain an optimality gap less than 4% with a reasonable primal residual. With the use of LTM, we hope to further reduce the computational time.

References

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