Benders Decomposition for Integrated Rolling Stock and Shunting Driver Rescheduling

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1. Introduction

Railway passenger operators normally use a sequential process to handle disruptions. First, the timetable is rescheduled according to the disruption. Based on this new timetable, the assignment of rolling stock to the trips is then rescheduled. The main goal in rolling stock rescheduling is to prevent any additional cancellations as a result of the disruption, while at the same time offering enough seats and minimizing the changes to the original rolling stock assignment. Lastly, a new crew schedule is made based on this new rolling stock assignment.

The decisions that are made in rolling stock rescheduling have a direct impact on the plans at the stations. Most importantly, the rolling stock assignment specifies at which stations the composition of a train is changed. To execute such changes, shunting actions are needed in which a train unit is driven from one location at the station to another. These shunting actions require resources at the stations, where we focus on the shunting drivers that are needed to drive these train units during shunting actions.

Currently, a sequential process is used to reschedule the rolling stock and the shunting drivers. In this process, one first fixes the set of allowed shunting actions and then solves the rolling stock rescheduling problem based on these possible actions. Next, it is determined at each station if a feasible plan can be made to accommodate the chosen shunting actions. If this is not the case, the set of shunting actions is updated and the process is repeated.

The disadvantage of this approach is that multiple iterations may be necessary to find a feasible solution, which might cost more time than is available in the rescheduling phase. Moreover, fixing the set of shunting actions beforehand might be overly restrictive, leading to rolling stock assignments of lower quality. To overcome these issues, we propose to solve the rolling stock rescheduling

problem jointly with the shunting driver rescheduling at the stations, a problem we refer to as the Integrated Rolling Stock and Shunting Driver Rescheduling Problem (IRSSDRP).

2. Problem Description

At the basis of the IRSSDRP is the timetable that is to be operated, where we assume that the timetable has already been updated for the disruption. Let S be the set of stations that are visited by the trips in the timetable. Moreover, we assume that transitions are known that determine how rolling stock moves from an incoming trip at a station to an outgoing trip. Let C be the set of these transitions and C_s the set of transitions at station $s \in S$.

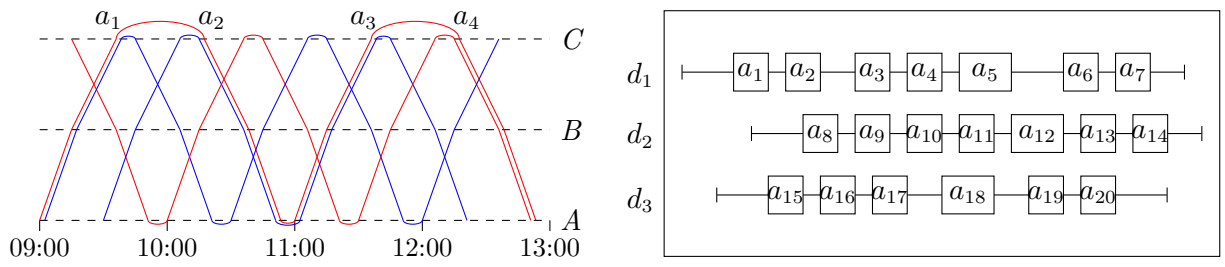
We assume that a heterogeneous fleet is available to operate the trips in the timetable. If two train units are of a compatible type, they can be combined into so-called compositions. A composition is thus a sequence of train unit types. Usually, only a subset of the compositions can be used on a trip, for example due to the maximum platform length that is encountered.

Compositions can be changed at the transitions. Let Q_c give the possible ways of changing a composition at transition $c \in C$, including those options in which the composition is not changed. Let $Q = \cup_{c \in C} Q_c$ then be the set of all composition changes. A composition change $q \in Q$ leads to tasks that need to be executed by shunting drivers. Let A_q be the set of tasks that need to be executed for composition change $q \in Q$. Each task describes from where to where a train unit needs to be moved and the times at which this shunting action starts and ends.

Rolling stock rescheduling amounts to assigning to each available train unit a sequence of trips that it executes during the day. These trip sequences for the units together imply compositions for the trips and composition changes for the transitions. In order for the trip sequences to be feasible, we need that all these compositions and composition changes are feasible for the trip or transition they are assigned to.

For each station $s \in S$, we also need to find duties for the drivers. Let D_s be the set of drivers that are available at station $s \in S$ and A_s the set of shunting tasks to be executed at this station. We then need to find a duty of the form $(a_1, \ldots, a_l)$ for each driver $d \in D_s$, where $a_1, \ldots, a_l \in A_s$. Together, the tasks in these duties need to cover all tasks in A_s.

In the Integrated Rolling Stock and Shunting Driver Rescheduling Problem (IRSSDRP), we need to find feasible trip sequences and duties under an objective function that balances the passenger service level with the operational costs. An example of a solution to the IRSSDRP is given in Figure 1. In the left figure, a rolling stock circulation is given that depicts the trip sequences for the train units of two rolling stock types, a blue and red type. The right figure then shows a duty assignment for one of the stations in the timetable, where the shunting actions are allocated to three shunting drivers. Note how shunting actions follow from composition changes at station C and that these shunting actions are allocated to the shunting drivers.



(a) Circulation

(b) Duty assignment for station C

Figure 1 **Example of a solution for the IRSSDRP. Given are a rolling stock circulation and duty assignment for one of the stations in the circulation.**

3. Methodology

To solve the IRSSDRP, we use a Benders Decomposition approach. In Benders Decomposition, we split the problem into a master problem and a subproblem. For each solution of the master problem, we find cuts by solving the subproblem and add these cuts to the master problem. This process is iterated until the master problem provides a solution for which no cuts can be generated in the subproblem.

Our master problem amounts to the rolling stock rescheduling problem. We solve this problem by means of the Composition Model as proposed by Fioole et al. (2006), which was used, among others, by Hoogervorst et al. (to appear) for the real-time rescheduling phase. Solving this model, which can be done by an off-the-shelf mixed integer programming (MIP) solver, gives us an assignment of compositions to the trips and of composition changes to the transitions. Let $Z_{c,q} \in \{0,1\}$ indicate whether composition change $q \in Q_c$ is chosen for transition $c \in C$.

For a given solution of the master problem, i.e., for $Z \in \{0,1\}^{|C|}$, we need to allocate duties to the drivers. This is done by solving for each station $s \in S$ the set-partitioning formulation

$$\min \sum_{d \in D_s} \sum_{k \in K_d} \omega_{d,k} Y_{d,k} \tag{1}$$

$$\text{s.t.} \sum_{d \in D_s} \sum_{k \in K_d} \kappa^d_{a,k} Y_{d,k} = \sum_{q \in Q_c : a \in A_q} Z_{c,q} \qquad \forall c \in C_s, a \in \bigcup_{q \in Q_c} A_q, \tag{2}$$

$$\sum_{k \in K_d} Y_{d,k} = 1 \qquad \forall d \in D_s, \tag{3}$$

$$Y_{d,k} \in \{0,1\} \qquad \forall d \in D_s, k \in K_d. \tag{4}$$

Here, K_d is the set of possible duties for driver $d \in D$, $\kappa^d_{a,k}$ indicates whether duty k contains task a and the binary variable $Y_{d,k}$ indicates whether driver d executes duty k. Constraints (2) ensure that all tasks are covered by a duty. Moreover, (3) ensures that a duty is assigned to each driver. As the number of possible duties is very large, we solve this model by means of column generation.

To solve the complete Benders Decomposition Model, we use a similar approach as in Cordeau et al. (2001). That implies that we relax the integrality restrictions on both the master and the

subproblem. In each iteration, we solve the relaxed master problem first and generate Benders cuts based on the current solution of the master problem. These Benders cuts are then added to the master problem. Moreover, branching decisions are made when a fractional solution is found, where we branch on variables in either the master or subproblem.

4. Computational Results

We apply our method to instances of Netherlands Railways (NS), which is the largest passenger railway operator in the Netherlands. The instances correspond to disruptions in which the railway infrastructure between two stations becomes blocked due to, e.g., an accident or technical failure. Our instances consider a large part of the Intercity network of NS and a blockage that occurs between the stations Utrecht Centraal and Driebergen-Zeist from either 7:00 to 10:00 or from 10:00 to 13:00. The results of our experiments are given in Table 1.

Table 1 Summary statistics of the performance of the Benders Decomposition Approach on the instances.

	7h - 10h	10h - 13h
Solving time (s)	152	23
Number of Benders cuts	46	44
Number of B&B nodes	152	42

The results in Table 1 show that our method is able to solve both instances within a few minutes and that the 10:00 to 13:00 instance can even be solved within half a minute. This difference between the instances is likely explained by the fact that our planning horizon resembles the remainder of the day, implying that a disruption at a later moment in time corresponds to a smaller planning horizon. Hence, each master problem can be solved more quickly and also the number of nodes to be explored is smaller when the disruption occurs later in time.

We also observe that while Benders cuts are needed to prevent infeasible solutions, the number of Benders cuts is relatively low. This is likely caused by the fact that only for a few stations the number of available shunting drivers is tight. Hence, most cuts will be generated for only a few of the stations. For example, for the 7:00 - 10:00 instance, 19 out of the 46 cuts are generated for only three of the stations in the timetable.

References

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