Robust Sort Plan Design for Express Parcel Delivery Systems

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1. Introduction

Recent years have brought significant changes in the operations of express delivery services, notably with the changing landscape of e-commerce worldwide. Parcel sortation systems, in particular, have become key components in such networks due to their ability to reduce per-unit handling and transportation costs through parcel consolidation. To be able to accommodate the increasing scale of e-commerce volumes as well as the rolling out of various service options, including same- and next-day deliveries, careful planning of parcel sortation systems is required to ensure that they remain cost effective without hindering the wider system goal of on-time delivery.

Service providers often operate different types of sortation systems that vary mainly in terms of layout design, equipment size and automation level. Common amongst their planning, however, are the operational constraints pertinent to resource capacity and time feasibility. Parcels arrive to the hub throughout the day, and they are required to be sorted and dispatched by specific deadlines. Failing to meet these deadlines may result in service failures, which in turn could hurt service provider's profitability and market position. In this context, we propose an optimization-based technology for creating cost-effective sort plans operated by express parcel delivery services. Unlike previous efforts in this area, we explicitly consider the various time-definite service guarantees offered in practice to generate plans that are time feasible from a service standpoint.

We focus on a two-stage sort system, as depicted in Figure 1. The operations start at receiving docks where parcels arrive to the hub in vans or trucks, and get unloaded and transferred to a cross-belt sortation system for primary sorting. Each parcel is then directed to a discharge point located at either side of the primary sort belt where it gets pushed via chute and grouped into what we refer to as a pre-sort pile. Piles that contain parcels with different destinations and/or deadlines are then moved using manually-operated carts to the secondary sort stage for detailed sorting. The goal is to generate sort plans that minimize sortation costs while maintaining operational and time feasibility under all possible demand realizations. In this paper, we focus on the commodity-to-pile assignment part of the problem, since its solution is directly impacted by the presence of

uncertainty in arrivals, especially, from a feasibility perspective. We introduce and test different worst-case mixed-integer programming formulations by considering various forms of uncertainty budgets modeled using polyhedral sets.

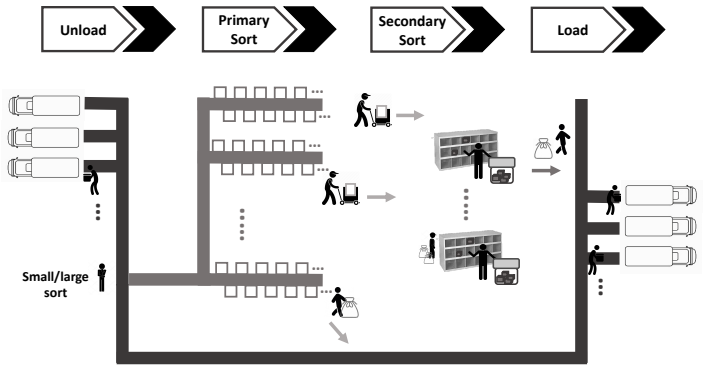


Figure 1 An illustration of a two-stage sort process

2. Approach

We introduce the *robust sort plan design* problem that aims in part to find a cost-effective configuration for primary sort equipment in the presence of demand uncertainty. We formulate the problem as a mixed-integer program and propose a robust optimization framework to solve it.

Let $\mathcal{T}$ be the set of equally spaced discrete time points that constitutes a sort shift. Furthermore, let $\mathcal{P}$ be the set of piles available in a primary sort area during a sort shift, and $\mathcal{C}$ be the set of potential pile cutoff times where $\mathcal{C} \subset \mathcal{T}$. Moreover, let μ_{ss} denote the secondary sort processing rate, measured in parcels per unit time. In addition, let $\mathcal{K}$ be the set of commodities arriving to the hub in a sort shift. Each commodity k has a cutoff time $d^k \in \mathcal{T}$, an arrival profile that includes arrival time $t \in \mathcal{T}$ and nominal arrival quantity q_t^k, and total nominal arrival quantity $q^k = \sum_{t \in T} q_t^k$, in parcels. To account for uncertainty in arrival quantities, we assume that the actual arrival quantity for each commodity $\hat{q}^k$ belongs to an interval-based uncertainty set $\mathcal{Q}$. We test three different representations of uncertainty sets, all of which limit the set of possible demand realizations to scaled deviation from nominal arrival quantities while keeping their arrival times unchanged. We denote such user-defined deviation by $\hat{\alpha} \in [0, 1]$. In what follows, we describe the three different robust formulations we consider and test in our study.

In model *RP1*, every commodity k has an arrival quantity $\hat{q}^k \in [q^k, q^k + \hat{\alpha} q^k]$. Total changes in arrival quantities from their nominal values are further restricted by a budget Γ, measured in commodities. In model *RP2*, every commodity k has an arrival quantity $\hat{q}^k \in [q^k - \hat{\alpha} q^k, q^k + \hat{\alpha} q^k]$. Total changes in arrival quantities from their nominal values are similarly restricted by a budget

Γ, measured in commodities. Total arrivals under all scenarios are also not allowed to exceed the nominal total arrivals $\sum_{k\in K} q^k$. Model *RP3* is just like *RP2*, except that the total changes in arrival quantities from their nominal values are restricted by a budget Γ, measured in parcels, and not in commodities.

In what follows, we present formulation *RP1* that corresponds to a reformulation of a bi-level deterministic mixed-integer program into a single-level robust counterpart using techniques from Bertsimas and Sim (2004). Models *RP2* and *RP3* are formulated in a similar fashion, but for brevity, we limit our presentation to *RP1* only. The primary decisions variables are $y_{p,c} \in \{1,0\}$ to indicate whether or not pile p with cutoff time c is used, and $x^k_{p,c} \in \{1,0\}$ to indicate whether or not commodity k is assigned to pile p with cutoff time c. Our goal is to generate a robust plan that minimizes the number of pre-sort piles requiring secondary sort as a proxy for minimizing the operational costs.

$$\underset{y,x}{\text{Minimize}} \quad \sum_{p\in\mathcal{P}}\sum_{c\in\mathcal{C}} y_{p,c} \tag{RP1-0}$$

$$\text{subject to} \quad \sum_{p\in\mathcal{P}}\sum_{c\in\mathcal{C}} x^k_{p,c} = 1 \qquad \forall k, \tag{RP1-1}$$

$$\sum_{k\in K}\sum_{t\le\tau\le c} q^k_\tau x^k_{p,c} + \Gamma z_{t,p,c} + \sum_{k\in K} w^k_{t,p,c} \le \mu_{ss}(c-t) \qquad \forall t,p,c \tag{RP1-2}$$

$$z_{t,p,c} + w^k_{t,p,c} \ge \sum_{t\le\tau\le c} \hat{\alpha} q^k_\tau x^k_{p,c} \qquad \forall k,t,p,c \tag{RP1-3}$$

$$\sum_{p\in\mathcal{P}}\sum_{c\in\mathcal{C}} y_{p,c} \le |\mathcal{P}|, \tag{RP1-4}$$

$$\sum_{k\in K} x^k_{p,c} \le n \qquad \forall p,c, \tag{RP1-5}$$

$$x^k_{p,c} \le y_{p,c} \qquad \forall k,p,c, \tag{RP1-6}$$

$$y_{p,c} \ge y_{p+1,c} \qquad \forall p,c. \tag{RP1-7}$$

Constraints (RP1-1) ensure that each commodity is assigned to a single pre-sort pile. Constraints (RP1-2)-(RP1-3) guarantee plan's time feasibility under the budgeted uncertainty. Constraint (RP1-4) ensures that the number of pre-sort pile positions that can be used is no more than the number of pre-sort pile positions available in the primary sort area. Constraints (RP1-5) place an upper bound on the number of commodities that can be assigned to a pile depending on secondary sort station configuration. Constraints (RP1-6) are coupling constraints and constraints (RP1-7) are added to break symmetry between pre-sort piles for computational efficiency.

3. Results

We test our approach using real-life instances obtained from a large service provider. We compare the performance of the three proposed robust formulations using various uncertainty budgets Γ.

Figure 2 illustrates the impact of the budget of uncertainty on the number of required secondary sort stations for one of the test instances. Introducing robustness clearly comes at the expense of requiring more secondary sort stations, thereby resulting in higher operational costs. For example, operating a sort plan generated by *RP1* with 100% protection level, i.e. when $\Gamma = |K|$, requires operating three additional manual stations when compared to the nominal plan requirements, i.e. plans generated with $\Gamma = 0$. Introducing robustness also impact the urgency of operations. Higher protection levels corresponds to more pre-sort piles with early deadlines as illustrated in Figure 3.

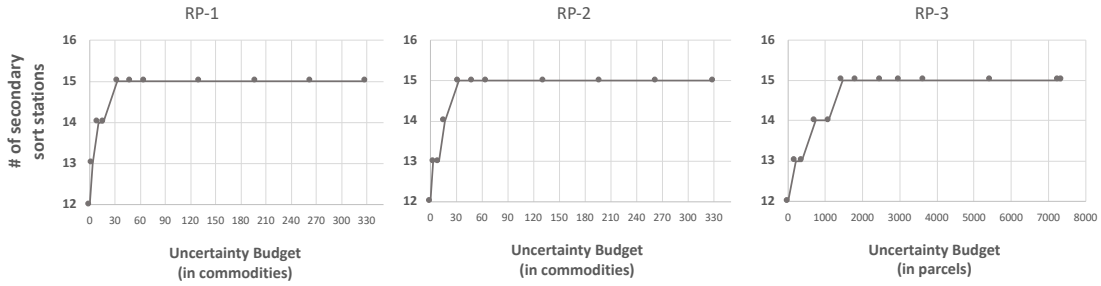


Figure 2 **Example showing the number of pre-sort piles that require secondary sorting as a function of the protection level Γ**

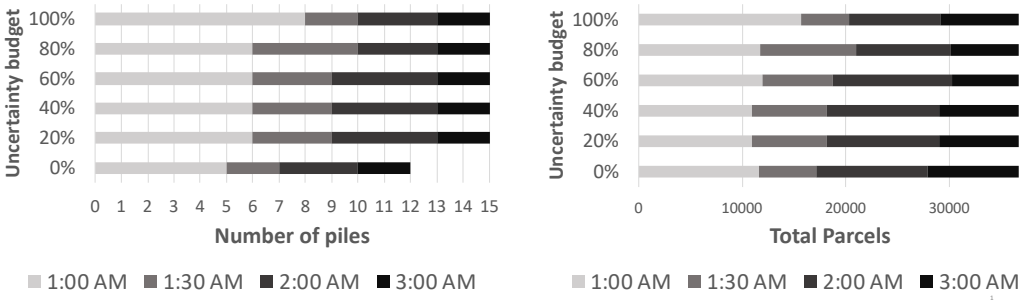


Figure 3 **Example showing the distribution of piles and parcels by deadline for different uncertainty budgets**

4. Conclusions

In this work, we proposed an optimization-based technology necessary for generating robust sort plans that are cost-effective and time feasible with respect to various service guarantees currently offered in practice. The proposed approach is practical not only for optimizing day-to-day operations in the presence of uncertainty, but also for gaining insights into the feasibility of decisions related to equipment sizing and resource requirement at the tactical level.

References

Bertsimas D, Sim M, 2004 *The price of robustness. Operations research* 52(1):35–53.