Bi-objective facility location in the presence of uncertainty

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1. Introduction

Many real world problems feature multiple and often conflicting objectives in an uncertain environment. Consequently, in practice, decision makers (DM) need to understand the trade-off between the considered objectives, considering different levels of uncertainty in order to choose a suitable solution. Although, many different approaches have been developed to cope with multiple objectives and uncertainty separately in optimization problems, the idea of intersecting these two domains, i.e. the analysis of decision problems involving multiple objectives and parameter uncertainty simultaneously, only received comparably little attention. Combinations of multi-objective optimization (MOO) with stochastic programming concepts (Gutjahr and Pichler (2016)) and robustness concepts (Ehrgott, Ide, and Schöbel (2014), Ide and Schöbel (2016)) give a flavor of the diverse possible ways to cope with real life decision making problems. Optimization problems arising along the humanitarian relief chain (HRC) motivated us in combining bi-objective optimization and optimization under uncertainty, as multiple objectives and inherent risk are distinguishing features of humanitarian logistics. As the number as well as the locations of the local distribution centers (LDCs) and their capacity directly affect the performance of the relief chain in terms of response time and costs (Balcik and Beamon (2008)), facility location decisions in disaster management are of vital importance. This paper deals with a two-stage bi-objective location-allocation problem for a last mile network where one objective is affected by data uncertainty. In most of the humanitarian literature, the expected value of a performance measure or worst-case robustness are considered to cope with uncertainty. In this paper, we analyze traditional two-stage stochastic and adaptive (two-stage) robust optimization as well as a risk-averse two-stage stochastic framework which can be seen as a method between the other two approaches using conditional value-at-risk (CVaR(α)). It can be adapted based on the DM's risk preference and uncertainty levels ($\alpha \in [0, 1)$). Two-stage stochastic and adaptive robust optimization are two special cases of the CVaR with $\alpha = 0$ and

$\alpha \approx 1$, respectively. Additionally, since large size instances are difficult to solve to optimality, we develop a matheuristic method which finds high-quality approximations of the Pareto-frontier.

2. Problem description

Due to scarcity of resources, in many disaster relief situations like slow-onset disasters such as droughts or sudden-onset disasters such as earthquakes, humanitarian organizations encounter challenging logistical *last mile* operations (Rancourt et al. (2015);Balcik and Beamon (2008)). On the one hand, the humanitarian organizations's goal in such a context is to reduce beneficiaries' vulnerability by maximizing coverage. On the other hand, they face limited economical resources and want to reduce their costs. Therefore DMs face two conflicting objectives and a trade-off solution needs to be identified. We consider a bi-objective location-allocation problem where the first objective is the minimization of location costs (efficiency-related criterion) and the second objective is to minimize the number of uncovered demand of beneficiaries (equity-related criterion). Moreover, as the demand of the affected people changes based on the crisis's intensity, we consider the amount of demand as an uncertain parameter in our model. We characterize the uncertainty in demand of beneficiaries via a finite discrete set of scenarios ($s \in S$ where $S = \{1, ..., N\}$). The problem is formulated on a graph $G = (V_0, A)$, where V_0 is the set of nodes and A is the set of arcs. V_0 can be partitioned into $\{\{0\}, V\}$, where $\{0\}$ is the main warehouse (MW), supplying the selected LDCs and V is the set of affected nodes and potential LDCs. We assume that the location of MW is determined before the optimization. Another assumption is that beneficiaries in each affected location ($i \in V$) will walk only to the one opened LDC ($j \in V$) which they are assigned to, in order to fulfill their demand ($q_i^{(s)}$) (single-source assumption). Moreover, beneficiaries will not walk to any LDCs, if their distance from an opened LDC is more than a certain distance (d_{max}). The first stage decision (y_j) is the location of LDCs with limited capacity (c_j) and given fixed cost (γ_j) that should be determined before the realization of the uncertain parameter. The second stage decisions are the assignments of demand nodes to the opened LDCs ($x_{ij}^{(s)}$) and the amount of relief items delivered to each LDC ($u_j^{(s)}$), which are determined based on first stage decisions and realized uncertain information.

3. Solution Procedure

As mentioned in the introduction, we embed three different uncertainty approaches, two risk measure methods, two-stage stochastic programming (expected value) and the two-stage CVaR approach, and adaptive (two-stage) robust optimization into two criterion space search frameworks, the well-known ϵ-constraint method due to its simplicity (Laumanns, Thiele, and Zitzler (2006)) and the recently developed balanced box method (Boland, Charkhgard, and Savelsbergh (2015)).

Moreover, we deploy two different variants of the CVaR model, the traditional CVaR and the subset-based CVaR in order to give an insight into the relation of different α-levels and the size of the required subset of scenarios (i.e. $\alpha = 1 - \frac{k}{N}$ when $\bar{S} \subseteq S, |\bar{S}| = k, |S| = N$). Therefore, different formulations of the base model are presented:

M1: Traditional two-stage stochastic bi-objective model

M2: Adaptive two-stage robust bi-objective model

M3-A: Two-stage CVaR bi-objective model

M3-B: Two-stage subset-based CVaR bi-objective model

We develop a decomposition method based on sorting the subsets of scenarios ($\bar{S} \subseteq S$) in order to solve the M3-B model faster.

Furthermore, in order to solve large scale instances close to optimality, we develop an iterative matheuristic method exploiting the identified solution of the Pareto-frontiers of the solved instances, where the cost of uncovered demand is the main objective and the cost of the opening LDCs (y_j) is the ϵ-constraint. In the first step, the first efficient solution is identified. Iteratively, all the other efficient solutions are then enumerated step by step until the last efficient solution with the minimum number of opened LDCs and maximum uncoverage cost is reached.

We observe that whenever we are able to solve an instance to optimality, very often one of the opened facilities (i.e. $y_j = 1$) is closed and the set of closed facilitates is kept unchanged (i.e. $y_j = 0$). However, there is a major difference between the first and the second solution along the frontier, not only in terms of the number of differences in y_j values (i.e. more than one open/closed facility), but also in terms of run time. Therefore, we develop the following procedure in order to find a high-quality approximation of the Pareto-frontier: (i) we solve the first efficient solution to optimality, (ii) we approximate the second efficient solution by solving the linear relaxation of the model and deploying local branching to speed up the algorithm, and (iii) we generate the rest of the Pareto-frontier iteratively by fixing variables of closed facilities (i.e. $y_j = 0$) derived from the previously enumerated solution.

4. Computational experiments

To analyze all the above-mentioned frameworks, we use real world data from the region of Thies in western Senegal (derived from Tricoire, Graf, and Gutjahr (2012)). These instances contain between 21 and 500 demand locations. For each instance, we consider 10 sample scenarios and time limit of 2 hours. Table 1 illustrates the number of solved instances using each model integrated into both ϵ-constraint (eps) and balanced box (BB) framework for different instances. We observe that the combination of the ϵ-constraint framework with adaptive robust optimization solves all of the instances to optimality and finds the complete Pareto frontier. The larger instances for smaller

α-levels cannot be solved within the time limit. The matheuristic method applied to M3-A and M3-B speeds up the bi-objective algorithms. Comparing the hypervolume indicators (Zitzler, Deb, and Thiele (2000)) shows that it is able to find high-quality approximations of the Pareto-frontier for those instances which the optimal frontier is known.

	eps-M1	BB-M1	eps-M2	BB-M2	eps-M3	BB-M3
#solved instances	16	12	28	15	13	12

Table 1 **number of solved instances for the six different approaches with the instance size 21 to 500**

We compare the solutions found by different approaches and use the expected value of perfect information (EVPI) and the value of stochastic solution (VSS) to show the benefit of using risk-measure models. The observations indicate that once the stochasticity has been taken into consideration, the solutions are more robust compare to deterministic average model.

5. Outlook

So far, we have considered only single source of uncertainty (uncertain demand). Our aim is to consider multiple sources of uncertainty (e.g. demand and road networks) in order to have a broader insight.

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