# Tabu Search for the Time-Dependent Vehicle Routing Problem with Time Windows on a Road Network

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**Motivation.** For more than 15 years now, it has been known that neglecting variations in travel times in cities, due, e.g., to congestion during peak hours, can lead to inefficient or even sometimes infeasible delivery routes (Ichoua, Gendreau, and Potvin 2003). Several authors have thus suggested time-dependent versions of the most commonly encountered vehicle routing problems. In these papers, however, time-dependent variations are usually defined with respect to customer-based graphs where the nodes are customers (and the depot) and an arc between two nodes corresponds to a fixed shortest path previously calculated in the underlying road network. See Gendreau, Ghiani, and Guerriero (2015) for an overview of the literature on time-dependent vehicle routing.

The Time-Dependent Vehicle Routing Problem with Time Windows on a Road Network ($TDVRPTW_{RN}$) is aimed at producing more realistic routes by taking into account the time of the day to compute the shortest path (in time) in a road network to travel from one customer to the next. That is, not only do we observe different travel times during the day to go from one customer to the next, but even the paths used are different. Considering different paths to travel from one customer to the next in the road network makes the problem much more complex.

Very few papers have addressed the $TDVRPTW_{RN}$ up to now: one may mention the heuristic of Mancini (2014) and the exact branch-and-price algorithm of Ben Ticha et al. (2019). The purpose of this talk is to present an efficient heuristic solution approach to solve the $TDVRPTW_{RN}$. This solution approach involves a tabu search metaheuristic that considers different shortest paths between any two customers at different times of the day. A major contribution of this work is the development of techniques to evaluate the feasibility as well as the approximate cost of a solution in constant time, which allows the overall solution approach to handle instances with up to 200 nodes and 580 arcs in very reasonable computing times.

**Problem Statement.** In the following, we first define the time-dependent road network graph. Then, we introduce two problems defined on this graph, namely, the Time-Dependent Shortest Path Problem ($TDSPP$) and the $TDVRPTW_{RN}$.

***Time-dependent road network.*** The road network is a directed graph $G = (V, E)$, where the set of vertices $V = \{0, 1, 2, ..., n\}$ corresponds to road junctions and the set of arcs $E$ to road segments between pairs of junctions. Each arc $(i, j)$ is associated with a distance $d_{ij}$, a time-dependent speed function $v_{ij} : t \rightarrow R^+$ that returns the speed at time $t$, and a time-dependent cost function $c_{ij} : t \rightarrow R^+$ that returns the cost of traversing arc $(i, j)$ at time $t$ (often, $c_{ij}$ corresponds to the travel time). Each speed function is a stepwise function, from which a piecewise travel time function can be derived.

***TDSPP.*** A path $p$ in the road network $G$ from a source node $s \in V$ to a destination node $s' \in V$ is defined as a sequence of consecutive arcs $(i_1, i_2), (i_2, i_3), ..., (i_{j-1}, i_j)$ with $i_1 = s$ and $i_j = s'$. Alternatively, the path can be viewed as the sequence of nodes $s = i_1, i_2, i_3, ..., i_{j-1}, i_j = s'$.

The $TDSPP$ then consists in identifying a minimum-cost path $p$ from a source node $i_1 = s$ to a destination node $i_j = s'$, given a departure time $t_0$. The cost of path $p$ at time $t_0$, $cp_p(t_0)$, is defined recursively using the time-dependent arc travel time functions. it should be noted that, for any departure time, the cost of a minimum-cost path from any source node to any destination node will be given by a piecewise linear function.

***TDVRPTW*** $_{RN}$. In the $TDVRPTW_{RN}$, a set of customers $C \subset V$ and a depot (node 0) are located on the time-dependent road network $G$, as defined above. Each customer $i \in C$ has a demand $q_i$, a time window for the service start time $tw_i = [a_i, b_i]$ and a service or dwell time $s_i$. A vehicle cannot arrive at customer $i$ after the upper bound $b_i$ of the time window, but can arrive before the lower bound $a_i$, in which case the vehicle waits until time $a_i$ to start the service. The set of vehicles $K$, each of capacity $Q$, is located at the depot. The time window at the depot $[a_0, b_0]$ defines the beginning and end of the time horizon. The problem is then to generate a set of feasible vehicle routes (solution), one for each vehicle, that start and end at the depot and serve all customers at minimum cost. The latter is obtained by summing the route durations (travel time + waiting time + service time), where the travel time is time-dependent. A solution is feasible if it satisfies the capacity constraints and the time windows.

Note that the travel time of a path in the road network between two customers $i$ and $j$ for any given departure time $t$, $tp_{i,j}(t)$, is a piecewise linear function derived from the speed functions of all arcs along the path.

**Approach.** Our solution approach combines two elements: an efficient algorithm for tackling the $TDSPP$ and a tabu search metaheuristic for solving the $TDVRPTW_{RN}$ proper.

***Solving the TDSPP.*** A time-dependent variant of Dijktra's algorithm is used to identify the minimum cost (travel time) path between a source customer $s$ and a target customer $s'$ in the road network $G$ at a given departure time $t_0$. At each iteration and starting from the current node, which is customer $s$ at the beginning, the arrival time at each non-permanently labeled successor is calculated with the IGP procedure (Ichoua, Gendreau, and Potvin 2003), using the speed function associated with that arc, to produce the arrival time at the successor. If the new path is better than the best known one, then the label is updated. In the special case where the successor is customer $s'$, the new path is considered only if $t_{s'}$ does not exceed the time window's upper bound of $s'$. Once all successors of the current vertex have been considered, the vertex in the road network with the minimum label among all those that are not permanently labeled becomes the current one and its label is made permanent. This is repeated until customer $s'$ is permanently labeled.

***Tabu search for the TDVRPTW***$_{RN}$. Our solution approach is based on the tabu search metaheuristic of Glover (1989), which has been widely used for solving vehicle routing problems. Starting from an initial solution generated with a simple heuristic, tabu search moves at each iteration to the best solution in the neighborhood of the current solution. To prevent cycling, it is forbidden to perform certain moves that could lead back to a previously visited solution. This is repeated until a stopping criterion is satisfied, at which point the best solution visited during the search is returned. In our case, the initial solution is produced with a greedy insertion heuristic and the neighborhood of the current solution is generated using CROSS exchanges (Taillard et al. 1997). Since evaluating this neighborhood is computationally expensive, different techniques are used to assess the feasibility and approximate the value of neighborhood solutions in constant time. Diversification is used in tabu search to favor the exploration of new regions in the solution space when search stagnation is observed. Diversification is realized by using an alternative objective, namely, minimization of the total distance, for a certain number of iterations. After a certain number of iterations with the distance objective, the original objective (duration) is restored until stagnation occurs again.

**Computational results.** We first describe the test instances used to perform our computational study. Then, a comparison is provided between our algorithm and the exact branch-and-price method $BP$ of Ben Ticha et al. (2019). All experiments were performed on a Dell PowerEdge R630 server with two Intel Xeon processors E5-2637V4 with 4 cores and 128GB of memory each.

***Test instances.*** The test instances, called $NEWLET$, come from Ben Ticha et al. (2019). Three graphs are available with $n = 50$, 100 and 200 nodes. Using various rules for deriving travel speed profiles and selecting customer nodes in the different networks, 21 categories of test instances are obtained. Finally, each category is duplicated by considering either narrow or wide time windows, for a total of 42 categories of instances. Since 5 different instances are available in each category, we have a total of 210 instances.

***Comparison with optimal solutions.*** It is important to note that the objective to be minimized in Ben Ticha et al. (2019) is the total distance. Thus, at the road network level, minimum-distance paths are considered between two customers, while the time-dependent travel times are only used to enforce path feasibility. We thus modified our algorithm by setting the total distance as the main objective and duration as the objective in the diversification phase.

Our computational results indicate that the gap between the final solutions produced by $TS$ and the optimum is under 1% on all instances, even if $TS$ was not intended to minimize the distance. The average computing times in seconds of $TS$ and $BP$ are 5.43 and 358.27 over the instances with narrow time windows, and 13.14 and 2454.46 for those with wide time windows. Thus, $TS$ is much faster on average, even after accounting for a 1.6 factor for processor speed difference.

***Minimum duration objective.*** Our algorithm was primarily designed to minimize the total duration of the routes, which seems appropriate in the case of time-dependent travel times. Even if there is no alternative algorithm to compare with, we ran our $TS$ on the 210 $NEWLET$ instances with this new objective (while minimizing the distance in the diversification phase). Detailed results will reported at the conference. When comparing the solutions produced by $TS$ when the distance or the duration objective is minimized, we observe that the solutions obtained are quite different.

**Conclusions.** We have proposed a tabu search heuristic, coupled with innovative techniques to evaluate neighborhood solutions in constant time, for the time-dependent vehicle routing problem with time windows on a road network. Computational experiments show that our method can identify high-quality solutions in very reasonable computation times.

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