Strategic Service Logistics Games with Customer-Induced Competition

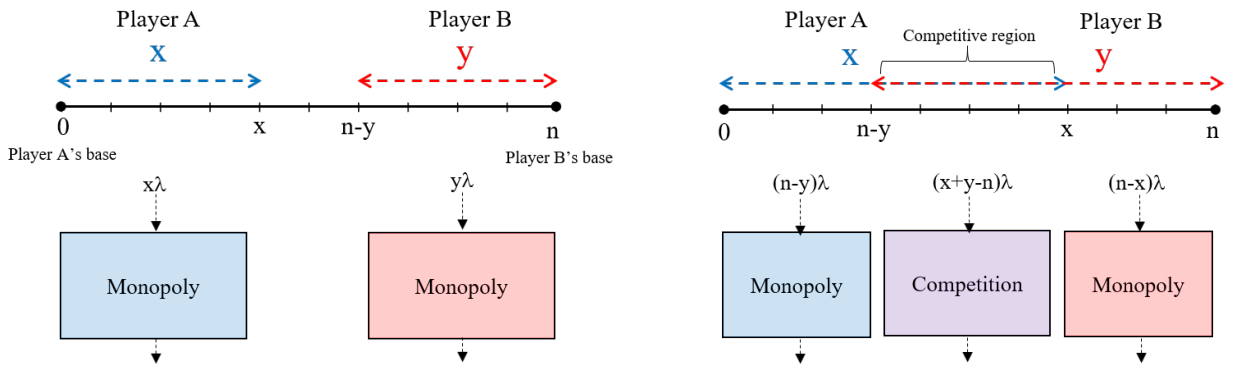
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1. Introduction

We study ambulance services or Emergency Medical Services (EMS) in emerging economies like Africa (Moh 2017, Kuo 2016), India (Seetharaman 2017) and South Korea. Here, centralized EMSs are in nascent stages of being established. Pre-existing ad-hoc decentralized services consisted of hospital networks that had established their own (small) fleets of ambulances, located in their service regions, to transport patients to hospitals. Often, the hospitals or clinics serve both as bases for the ambulances as well as the destination that patients need to be transported to. Callers hedge in emergencies by calling more than one ad-hoc and/or the centralized service provider, hoping to receive service from the fastest one that arrives on-scene. Patients depart with the earlier arriving ambulance, leaving the slower-arriving service provider to 'discover' that she has been abandoned. Because service providers' resources are temporally occupied, even to discover whether they have lost or won the customer, the later-arriving ambulance not only loses revenue but also incurs an opportunity cost of not being available to serve other potential demand occurring during its travel time; further stressing an already resource-constrained system.

Service providers, being cognizant of such customer behavior, can appropriately and strategically define their service regions to allow for an optimal trade-off for themselves, or dynamically choose whether or not to respond to certain customers. Our goal in this paper is to *study the service providers' strategic behavior* in choosing their coverage areas where customers make multiple orders and to analyze if customers are better off by making duplicate orders.

To the extent of our knowledge, this is the first work that formally studies a *competitive setting with temporal aspects of server availability*, and provides insights into effective strategies for service providers. On one hand, competition of a purely temporal nature in queues has been captured without the spatial aspect (Hassin and Haviv 2003, Allon and Federgruen 2008). On the other hand, the Hotelling's location model (Hotelling 1929) and its many variations study spatial competition (e.g., Brenner (2011)), to locate facilities under competition but do not explicitly account for servers' temporal availability. These considerations become important in service settings with spatial context, as opposed to product settings, where the existence of a facility is not as important as the *availability of a server* at the facility. Our work therefore focuses not on product capacity (availability of facility) but on associated service considerations, in particular, lost demand.



(a) Two independent basic queuing models without coverage overlap.

(b) When coverage areas overlap, the behaviors of players are not independent.

Figure 1 **Competitive players locate on either end of the service region and each tries to find a coverage range to maximize her own utility given the other player's range.**

2. Problem Statement

We consider a simplified version of this problem (see Figure 1), where two service providers/players A and B, offering identical service, are located at either endpoints of a unit line partitioned into n equal segments. Each provider operates a single server from their base of operations, and functions as a loss system. Customers arrive at the midpoints of a segment, uniformly distributed among the n segments, according to a temporal Poisson process with rate λ. The service rate is location-dependent. Specifically, for a region at a distance d from the server's base, the server's service time is exponentially distributed with mean $[(d-1)+d]/(2n\mu)$. Because customers can call multiple service providers, providers strategically determine the boundary of their service regions to maximize their utility, accounting for the stochasticity, wasted attempts and opportunity costs. Specifically, service provider A (or B) will determine her service region to be x (or y) segments from her base location. If these service (coverage) areas overlap, customers in the overlap call both players, and accept service from the first-arriving server. We define a server's utility as the number of customers it serves. (Alternative utility functions are discussed in the full paper.) We study the existence and nature of equilibria, equilibrium coverage ranges, price of anarchy and discuss managerial insights.

3. Approach

3.1. Embedded Markov Chain of the Joint System

The system dynamics regarding the availability of each player and probability of arriving first (if both servers are available) are important to formulate the utility. Since we assume a Poisson arrival process and exponential location-dependent service times, the joint system can be described as an embedded Markov chain (EMC). The state space is defined as $\mathcal{S} = \{(i,j) : i \in (0,\ldots,x),\ j \in (0,\ldots,y)\}$, where i and j represent if it is idle $(i,j=0)$ or the customer locations $(i,j>0)$ that players A and B are serving respectively. Thus, the system has a stationary distribution. That is,

a customer arriving in A's coverage area $[1,x]$ finds A's server idle (available to serve) with some probability $P_A(x) = P(s \in (0,*) \subset \mathcal{S})$. Similarly, $P_B(y) = P(s \in (*,0) \subset \mathcal{S})$. This implies that the stationary probability of each player's availability can be calculated using the Erlang-B formula. $P_{AB}(x,y) = P(s \in (0,0) \subset \mathcal{S})$, the stationary probability that both players are idle simultaneously, is coupled. We model the joint idle probability in analytical form using alternating renewal processes and compute an exact analytical form that includes an inverse of the EMC's transition probability matrix. As this is too complex to compute for arbitrary x,y and n, closed form equilbrium results are challenging. Our renewal-theoretic analysis also points to a simpler, more tractable approximation for the stationary probability using simplified embedded Markov chain which has only 4 states, $\mathcal{S}' = \{(i,j) : i \in \{0,1\},\ j \in \{0,1\}\}$. Using this simplified Markov chain for equilibrium analysis, we show the validity of the approximation and the existence of equilibrium.

3.2. Utility function formulation

We build utility function using the stationary probability obtained by the EMC. The utility function is dependent on the choice of service range. If providers' service regions overlap, creating competition, their utility should be less than that of from monopoly market.

DEFINITION 1. Assuming A has a monopoly over her coverage region x, the 'monopolistic utility' $U_M^A(x)$ of player A is:

$$U_M^A(x) = \lambda \frac{x}{n} P_A(x) \tag{1}$$

If competitive region exists, i.e., $x > n - y$, 'competitive utility' $U_C^A(x,y)$ of player A is:

$$U_C^A(x,y) = U_M^A(x) - L^A(x,y), \text{ where } L^A(x,y) = \lambda \frac{x^2 - (n-y)^2}{2n^2} P_{AB}(x,y) \tag{2}$$

4. Results

Lemma 1 *For any fixed $y \in [1,n]$, both the monopolistic utility $U_M^A(x)$ (and symmetrically, $U_M^B(y)$)and the competitive utility $U_C^A(x,y)$ (and $U_C^B(x,y)$) are unimodal.*

Lemma 2 *Suppose $x_0 = \arg\max_x U_C^A(x,y)$ for any fixed $y \in [1,n]$ and a pure strategy Nash equilibrium (x^*, y^*) exist. Then, the equilibrium coverage ranges $x^*, y^* \le x_0$.*

4.1. Equilibrium Analysis

A symmetric Pure Strategy Nash Equilibrium (PSNE) exists for all utility functions described.

Theorem 1 *Let n be the number of points in the service region and assume the discritized service region includes the point $n/2$. Then, there exists a symmetric pure strategy Nash equilibrium for all $\rho > 0$. The symmetric PSNE $x^* = y^*$ occurs at:*

$$x^* \in \begin{cases} \lfloor n/\sqrt{\rho} \rfloor \text{ or } \lceil n/\sqrt{\rho} \rceil, & \text{if } \rho \ge 4 \\ \frac{n}{2}, & \text{if } 2 \le \rho < 4 \\ \{\frac{n}{2}, \ldots, \lceil n/\sqrt{\rho} \rceil\}, & \text{if } \rho < 2 \end{cases}$$

Theorem 1 shows that providers have symmetric PSNE with overlapping coverage regions when $\rho < 2$ (relatively low demand). When the ratio of the service demand and supply exceeds some level, $\rho \geq 2$, customers have at most one available provider, thus can have at most single order.

Policy	Total Utility	Blocking Probability	Expected Service Time
symmetric PSNE	0.8406	0.1092	0.3050
coco	0.8888	0.1111	0.3750
Centralized Dispatch	0.9596	0.0400	0.4010

Table 1 Outcomes of symmetric PSNE, coco, and centralized dispatch.

4.2. Loss of Efficiency in the Competitive Logistic Model

For cases where Theorem 1 points to equilibrium with overlapping coverage ranges, EMS providers can waste both operating costs for service attempts and opportunity costs of missed customers. Such inefficiencies are due to customers' selfish behavior when they attempt to secure the fastest service. Would this selfish customer better off themselves by making duplicate orders? Yes and no. Suppose the system uses a 'centralized dispatch' policy where a centralized call center assigns each order to the nearest available server. Note that closest server is not always the fastest one because the service time is exponentially distributed. Then, the overall rate of successful service is increased because the blocking probability is smaller without missed customers. However, the customer's expected service time increases. Thus, if each customer makes a single order, they would have higher probability of being served, but with a higher expected service time conditional on being served. This points to the inherent tradeoff in the system from the providers' and customers' perspectives. We also discuss the coco solution, which is a bargaining solution that lies between the fully centralized and decentralized operations.

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