On-demand Urban Aerial Mobility Planning: An Adaptive Discretization Approach

Kai Wang
Sloan School of Management, Massachusetts Institute of Technology, Cambridge, Massachusetts; Email: cwangkai@mit.edu,

Alexandre Jacquillat
Sloan School of Management, Massachusetts Institute of Technology, Cambridge, Massachusetts; Email: alexjacq@mit.edu,

Vikrant Vaze
Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire; Email: vikrant.s.vaze@dartmouth.edu,

---

Most major cities across the world face critical challenges to accommodate growing travel demand with limited infrastructure. As urban population, mobility needs and e-commerce deliveries grow rapidly, so do congestion, energy use, and greenhouse gas emissions. Urban transportation is being transformed by on-demand businesses (e.g., car-sharing, ride-sharing) and new vehicle technologies (e.g., electric, connected, autonomous vehicles). But long-term sustainable growth of urban transportation may require novel technology, business and policy solutions.

New opportunities are opening due to a prominent science fiction topic becoming reality: the rise of electric vertical-takeoff-and-landing (eVTOL) vehicles. Figure 1 shows a concept from Uber Elevate and sample prototypes from Boeing and Airbus. These vehicles rely on lift propellers for vertical take-off and landing as well as a cruise propeller and wings for en-route cruise. Particular features of eVTOL vehicles are that: (i) they take off and land on limited infrastructure rather than full-length runways; (ii) they have a small payload that can accommodate a few passengers, (iii) they fly short-range operations, and (iv) they are electrically powered by batteries. As a result, eVTOL technologies can provide efficient urban transportation with limited contributions to greenhouse gas emissions.

The development of eVTOL vehicles gives rise to a new form of transportation: *Urban Aerial Mobility (UAM)*. Early UAM developments include Uber Copter, which uses helicopters to fly between Manhattan and JFK Airport; pilots of flying taxis in Dubai and Singapore; and UAM testing by NASA and Uber. Despite many uncertainties, a few operating characteristics are emerging. First, UAM relies on on-demand operations with small-payload vehicles—as opposed to scheduled operations with larger aircraft as in traditional aviation. Second, UAM relies on centralized



(a) Uber Elevate (b) Boeing Aurora PAV (c) Airbus Vahana

Figure 1 Sample eVTOL prototypes.

passenger-vehicle matching and passenger pooling—as opposed to customers choosing their vehicles and driving on their own as in car-sharing. Third, UAM relies on a dedicated infrastructure of *vertiports* for vehicles to take off and land—as opposed to ride-hailing.

As a first step toward the optimization of on-demand UAM systems, this paper focuses on the planning of vertiport networks. Specifically, it optimizes the number, location and capacity of vertiports in a metropolitan area—one of the prominent strategic challenges currently faced by UAM operators. The core trade-off involves deploying many scattered vertiports with small capacity to cover hotspots of customer demand vs. deploying a few central vertiports with high capacity to facilitate operations (e.g., passenger pooling, vehicle routing).

This paper first formulates an integrated model of vertiport deployment. The proposed model captures two critical complexities. First, there exist interdependencies between vertiport deployment at the strategic level and UAM operations at the tactical level. To capture these, the model uses a queuing network to replicate passenger-vehicle matching, passenger pooling, battery recharging, vehicle routing and repositioning. Second, there exist interdependencies between supply-side UAM operations and demand-side customer adoption. To capture these, the model embeds a customer adoption function that increases with level of service.

Our model is formulated as a mixed-integer nonlinear program, of the following form:

$$\begin{aligned} \min_{\boldsymbol{u},\boldsymbol{\alpha},\tilde{\boldsymbol{\tau}}} \quad & \boldsymbol{c}^T\boldsymbol{u} \\ \text{s.t.} \quad & \boldsymbol{F}\boldsymbol{u}+\boldsymbol{G}\boldsymbol{\alpha}+\boldsymbol{H}\tilde{\boldsymbol{\tau}}=\boldsymbol{g} \\ & \|\boldsymbol{A}_k\boldsymbol{u}+b_k\|_2 \le \boldsymbol{e}_k^T\boldsymbol{u}+d_k, \quad \forall k=1,\cdots,n \\ & \boldsymbol{\alpha} \le f(\tilde{\boldsymbol{\tau}}) \end{aligned}$$

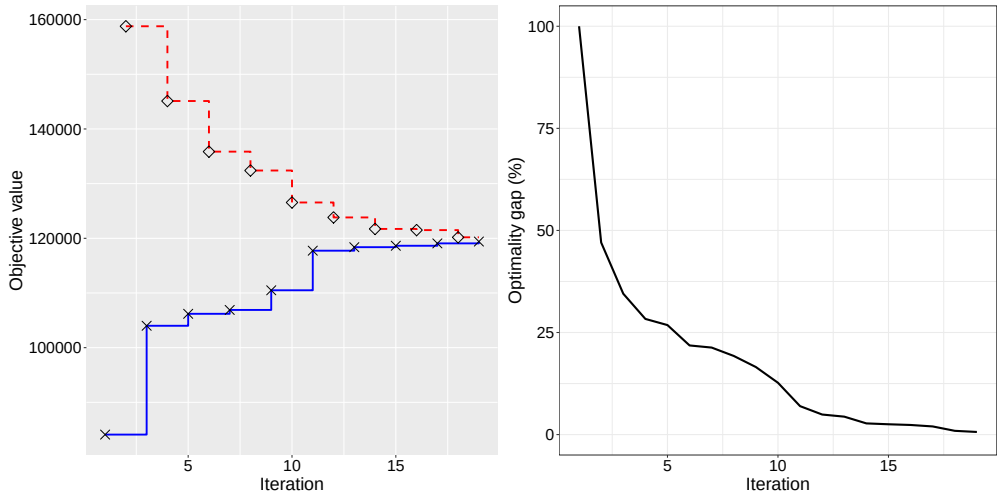
The model includes three classes of constraints: linear constraints, second-order conic constraints, and more general nonlinear constraints (coming from the customer adoption model). The first two sets of constraints lead to a second-order conic program—which remains tractable with modern optimization solvers. But the customer adoption constraints assume a more general form $\boldsymbol{\alpha} \le f(\tilde{\boldsymbol{\tau}})$,

where $\tilde{\boldsymbol{\tau}}$ denotes the level of service and $\boldsymbol{\alpha}$ denotes customer adoption. The function $f(\cdot)$ can be obtained from historical data, from analytical demand models, or a combination thereof. Given the uncertainty surrounding UAM adoption, this paper uses chance constraints embedded with distributionally robust optimization. We only assume that the function $f(\cdot)$ is (strictly) monotonic, reflecting that a higher level of service leads to higher customer adoption. But the function $f(\cdot)$ can be nonlinear and non-convex—thus making the model very hard to solve.

To solve this nonlinear problem, this paper then develops an original solution algorithm based on adaptive discretization. This algorithm can be applied to a range of models combining a "tractable part" (here, a mixed-integer second-order conic program) and general constraints of the form $\boldsymbol{\alpha} \leq f(\tilde{\boldsymbol{\tau}})$ (where $f(\cdot)$ can be any strictly monotonic function). The algorithm iterates between a *conservative model* that provides a feasible solution and a *relaxed model* that provides a solution guarantee. Both models have the same complexity as the "tractable part" of the overall model, and both models approximate the function $f(\cdot)$ by piece-wise constants or piece-wise linear segments (leveraging partial convexity). The algorithm alternates between the two models, iteratively updating a lower bound and an upper bound of the optimal solution. We propose an adaptive discretization procedure to enlarge the conservative model's feasible region (by exploiting good solutions) and to reduce the relaxed model's (to avoid local optima). Ultimately, the proposed adaptive discretization algorithm provides a computationally tractable solution approach with provable quality guarantees to a broad class of nonlinear optimization problems.

Computational results of this paper demonstrate that the proposed algorithm provides high-quality solutions for real-world networks. Using data from New York City, experimental results in Figure 2 and Table 1 show that the algorithm yields near-optimal solutions in around 20 iterations and reasonable computational times. Specifically, the algorithm provides two key benefits. First, the algorithm provides provable solution quality guarantees (see Figure 2): by comparing the lower bound and the upper bound, our solution is known to lie within 1% of the true optimum. Second, it exhibits superior properties than baseline approaches based on static discretization (see Table 1). As compared to coarse discretization, our adaptive discretization algorithm yields much better solutions—albeit in slightly longer computational times; and as compared to granular discretization, our adaptive discretization algorithm yields better *and* faster solutions.

The paper concludes with managerial insights on the structure of optimal UAM networks. We find that the optimal UAM network is not dense, with 3–10 vertiports serving 13–81 origin-destination pairs in New York City. The UAM system predominantly serves origin-destination pairs with high demand and long travel times, where it provides stronger benefits compared to ground transportation. Our results also shed light on two interesting patterns. First, vertiports are not necessarily constructed in areas of highest demand—to balance low travel times when customers are served vs.



(a) Lower and upper bounds (b) Optimality gap

Figure 2 Convergence of the adaptive discretization algorithm.

Table 1 Comparison of our algorithm and static discretization, as a function of the demand

Method	Demand	$D=1.5$	$D=2$	$D=3$	$D=4$	$D=5$	$D=6$	$D=7$
Our algorithm	lower bound (LB)	2,258.23	9,193.36	24,454.37	44,825.25	64,669.06	87,440.94	119,409.31
	upper bound (UB)	2,272.81	9,266.68	24,599.51	45,120.56	65,005.24	87,528.28	120,182.92
	iterations	19	17	13	15	21	25	19
	time (minutes)	1.06	1.15	2.48	100.91	335.47	658.33	800.59
Coarse discretization	objective	0.00	6,417.80	19,684.81	40,846.54	59,279.72	78,844.14	98,932.93
	gap from LB	100.00%	30.19%	19.50%	8.88%	8.33%	9.83%	17.15%
	time (minutes)	0.26	0.27	0.38	1.79	16.63	18.24	37.76
Granular discretization	objective	1,780.22	8,901.27	24,028.62	43,954.06	63,886.33	83,087.07	108,965.15
	gap from LB	21.17%	**3.18%**	**1.74%**	**1.94%**	**1.21%**	**4.98%**	**8.75%**
	time (minutes)	0.58	**1.87**	**30.71**	**332.56**	**638.07**	**>1,440**	**>1,440**

mitigate the inconvenience for customers that are lost due to service unreliability. Second, repositioning trips can account for more than 50% of all trips—because of the interplay between battery recharging, vertiport capacities, and service reliability. Most importantly, the optimal vertiport network grows in a nested fashion: as UAM penetration increases, the model incrementally adds new vertiports rather than completely changing the network of vertiports. This suggests opportunities to deploy a phased approach in practice, starting with a few "obvious" vertiports and then expanding the network as UAM technologies mature.