Transit user route-choice modeling with risk-preference

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The reliability of transit routes, especially in urban areas, is mostly affected by recurrent and non-recurrent congestion leading to high variations and uncertainties in route travel time. Several studies on the reliability and its impact on route choice have been done. However, the perspective is important, especially for sensitive travelers. Although several passenger routing models exist, an approach that addresses the risk-preference of travelers due to transit network uncertainties has not been developed. If travelers are assumed not to be perfectly rational, then their perspective to risk and uncertainties should affect the selection of strategies when navigating in transit networks. We need a new approach to select strategies in the transit path choice by assuming travelers are sensitive to uncertainties. Hyperpaths have been widely studied and developed for providing routing options for travelers using public transit (Nguyen, Pallottino, and Gendreau., 1998; Verbas, Mahmassani, and Hyland., 2015; Opasanon and Miller-Hooks., 2001). To address a travelers' perspective to uncertainties, we propose a modified generalized cost for evaluating hyperpaths that reflects the risk-preference of a traveler.

The perspective of a traveler to variabilities in transit networks can be characterized through the exponential utility as:

$$u^{\lambda}(\tau) = -(sgn(\lambda))e^{-\lambda\tau}. \tag{1}$$

For values of $\lambda > 0$,

$$u'(\tau) = \frac{du}{d\tau} = \lambda e^{-\lambda\tau} > 0, \text{ and } u''(\tau) = \frac{d^2u}{d\tau^2} = -\lambda^2 e^{-\lambda\tau} < 0.$$

This function is increasing and concave. The local measure of risk-aversion, known as the *Arrow-pratt measure of absolute risk aversion* at τ is given as $\frac{-u''(\tau)}{u'(\tau)} = \lambda$ implying the greater the value of λ, the more risk-averse the traveler is.

Uncertainties in on-time arrival of bus r at origin stop i (delays, a_i^r) and in-vehicle travel time ($IVTT$) of a path can therefore be transformed through the utility function in Equation (1) to reflect a travelers perspective to uncertain outcomes. If a traveler is risk-averse, then it is logical for

them to follow the strategies generated through the concave exponential utility function. Travelers will value their choices according to the value function resulting in selections of more certain path but possibly higher total cost. Consider an acyclic graph $G = (I, L)$, where $i \in I$ and $l \in L$ is the set of stops and links in the transit network respectively. If $IVTT$ for routes or links l is described through a normal distribution $v(\mu_l, \sigma_l^2)$ and the delays or actual departure times a_i^r at origin stop i for run r(measured with the scheduled departure time as reference point) is characterized through a log-normal distribution $d(\mu_i, \sigma_i^2)^r$, then similar to the random network formulation presented by Polychronopoulos and Tsitsiklis (1996), the total $IVTT$ of a path $P \subset L$ is the sum of link or route traversal time forming P, each of which are assumed to be normally distributed. Thus, the total $IVTT$ of path P is selected from $v(\mu_P, \sigma_P^2)$, where $\mu_P = \sum_P \mu_l$ and $\sigma_P^2 = \sum_P \sigma_l^2$. For any given destination, a preferred arrival time (PAT) and time window dt, we generate a hyperpath or strategy from transit schedule data that specifies a set of attractive lines for every stop using a backward path search and trip based hyperpath approach (Khani., 2012). The goal of the risk-sensitive traveler is to find the origin stop run(departure time) and path in the acyclic graph which has minimum risk. We propose a new risk-averse generalized cost ($RAGC$) of traveling from stop i using run r to destination through Path P as

$$RAGC_{P_i}^r = EU_{P_i}^{a_i^r} + EU_{P_i}^{IVTT} + \alpha_1 W_{P_i} + \alpha_2 WT_{P_i} + \alpha_3 T_{P_i}. \tag{2}$$

$EU_{P_i}^{{a_i}^r}$ and $EU_{P_i}^{IVTT}$ are the expected utility for a_i^r and $IVTT$ for path P from origin stop i and W, WT, and T is the total waiting time, walking time and number of transfers T respectively for using path P weighted by α_1, α_2 and α_3 respectively. By taking the negative of W, WT, T, $IVTT$, a_i^r, the general objective is to find the path and run that maximizes $RAGC$ from a given origin stop i.

$$\max_{P_i \in P} RAGC_{P_i}^r. \tag{3}$$

The total expected utility for $IVTT$ to reach destination from origin stop i using path P is written as

$$EU_{P_i}^{IVTT} = \int u^\lambda(IVTT) dV(IVTT) = \int v(IVTT) u^\lambda(IVTT) dIVTT, \tag{4}$$

where $V(IVTT)$ and $v(IVTT)$ are cumulative distribution and probability density functions. Assuming buses are instructed to wait at stops when arrival time is less than scheduled departure time (SDT), then a normally distributed $IVTT$ between two consecutive stops will mostly lead to a log-normally distributed delays a_i^r or actual departure time for run r at stop i (potentially right skewed). The probability density function is written as

$$d(\mu_i, \sigma_i^2)^r = \frac{1}{\sqrt{2\pi}\sigma a_i^r} e^{-\frac{1}{2}\left(\frac{\ln(a_i^r)-\mu}{\sigma}\right)^2}. \tag{5}$$

Given the delay (actual departure time) distribution $d(\mu_i, \sigma_i)^r$, the expected utility for various alternative attractive runs r at stop i is given as

$$EU_{P_i}^{a_i^r} = \int u^{\lambda}(a_i^r) dD(a_i^r) = \int d(a_i^r) u^{\lambda}(a_i^r) da, \tag{6}$$

where $D(a_i^r)$ and $d(a_i^r)$ are cumulative distribution and probability density function.

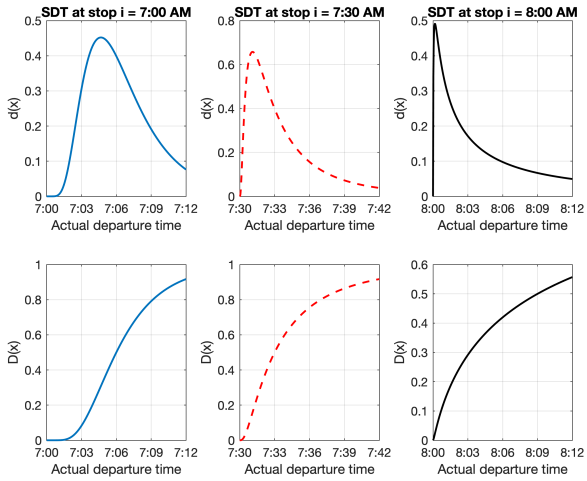


Figure 1 **Delay (actual departure time delay) distribution at origin stop i assuming buses must wait at stop if arrival time at stop i is less than the SDT at stop i.**

Assuming a destination stop 10, $PAT = 6:30$, $dt = 15$ min, $\lambda = 0.5$, and $\alpha_1 = \alpha_2 = \alpha_3 = 0.33$, figure 2 describes the results of transit network having three routes, 10 stops and 2 transfer stops (4,6). Each route has a total of 4 trips. WT between stops 4-5, 5-6, and 6-7 are assumed to be 10 min each. Waiting time at transfer stops is calculated by subtracting the scheduled arrival time of traveler at a stop from the scheduled departure time of attractive run at that stop. $IVTT$ for each path in the hyperpath was represented by a normal distribution whose mean is equal to the total travel time for that path between origin stop and destination calculated from schedule data. Variance for each path was randomly selected from a uniform distribution between {3, 6, 9}. a_i^r was represented by a log-normal distribution with mean equal to the log(2), and variance randomly selected from a uniform distribution between {0.5, 1, 2}. The resulting hyperpath and calculated $RAGC$ is shown in Figure 2. It is important to note that this formulation works well when assessing risk-preference for variables characterized by uncertainties (e.g., a_i^r and $IVTT$) as such adding T, WT, and W had an insignificant contribution to the overall cost.

In this work, we model the perspective of a traveler in select minimum risk strategies at a given origin stop. As an extension, a learning-based approach incorporating a multi-modal framework will be presented. A large scale analysis that considers different levels of sensitivity (λ) will also be evaluated.

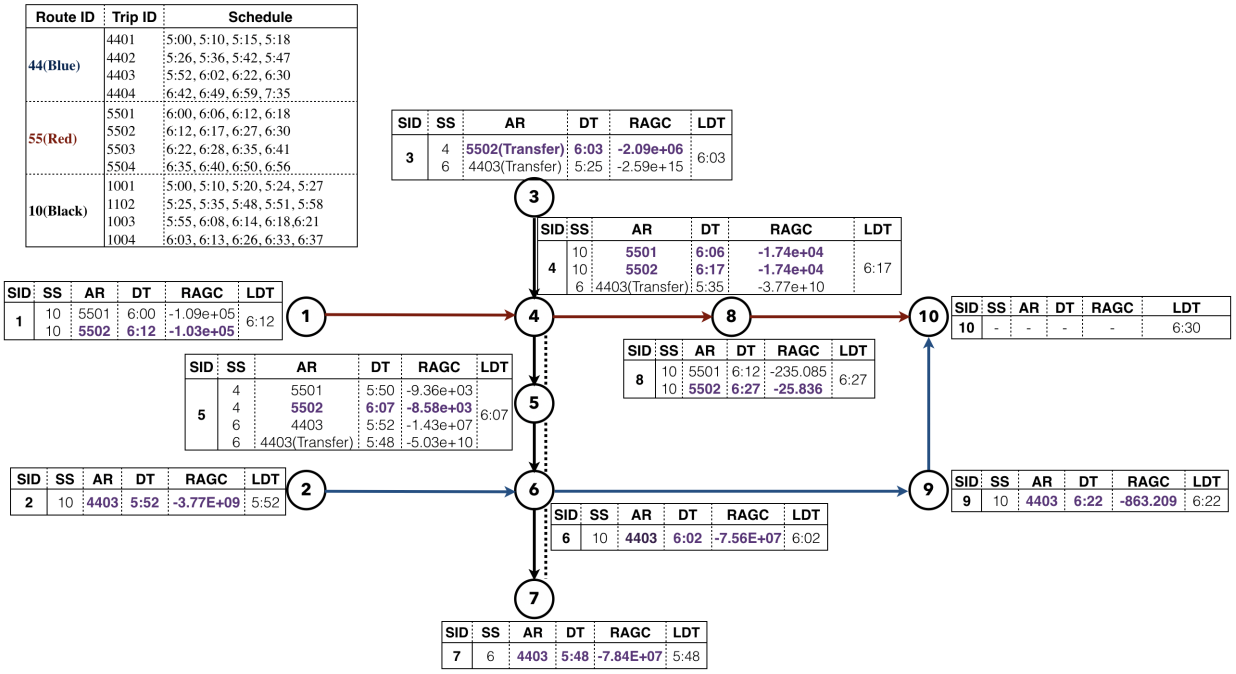


Figure 2 **Results of Hyperpath with RAGC for selecting minimum risk path at any given stop. SID: Stop ID, SS: Successor Stop, AR: Attractive Run, DT: Departure Time, RAGC: Risk-Averse General Cost, LDT: Latest Departure Time to ensure arrival within PAT. Bold-faced violet text in the figure shows run (strategy) that will be recommended for a risk-averse traveler with $\lambda = 0.5$**

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