A Pick up and Delivery Problem with Demand Regions

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Motivation Less than truckload (LTL) shipping involves the transportation of freight weighing between 150 and nearly 20,000 pounds (roughly). Typically, LTL shipments from multiple customers are pooled into full truckload shipments assigned to carriers. The paper deals with this logistics environment. Not all origins and destinations are created equal. In many industrial vehicle routing situations, the origin and destination of a route is not located at a depot, but rather within a road network. Ride sharing provides an instructive example of such a setting. In such a situation, it is desirable for the endpoint of a route to be in an area of high demand, enabling the vehicle to quickly begin another route without spending much time empty. Conversely, it is undesirable for a vehicle to complete its route in an area of low demand, as this diminishes the likelihood of a new request appearing nearby, forcing a vehicle to travel empty towards an area of high demand. This preference is heightened by the ability of ride sharing companies to charge a premium for requests originating in high demand areas. The significant effect of the demand distribution in ride sharing settings was studied in Bimpikis, Candogan, and Saban (2019). Another setting where such considerations are relevant is the situation of a routing company which utilizes a contract with a delivery company, which charges the routing company based on the route design. In this situation, too, it would be desirable for the routing company to develop routes ending in high-demand areas, as contractors will charge a premium for routes terminating in low-demand areas. This is the setting presented to us. We consider a problem designed to solve the routing problem in this situation, which captures preferences for high demand regions in the objective function. In this problem, the metric of total distance traveled continues to be instrumental, but is no longer the lone decision-driving force. It may indeed be preferential for a vehicle to travel a little farther in order for their route to terminate in an area of high demand.

Problem Statement Ours is a variation of the well-studied pick up and delivery problem (for a broad account, see Parragh, Doerner, and Hartl (2008)), with a few key differences. Consider a

set of customers, each represented by a pick-up location and a delivery location. Each node in the network is associated with a demand level. We make a number of assumptions, provided by our industry contact, that define our problem. First, we assume a limitless fleet of vehicles dispersed throughout the network, allowing routes to originate at any pickup location at no cost to the objective. This assumption can also reflect an abundance of available cars in a ride sharing setting (an increasingly realistic premise). Furthermore, we assume no fixed cost is associated with the use of a vehicle. A route must begin at the pick-up location of a customer and must end at the delivery location of a customer (though these need not be associated with the same customer). Precedence constraints must be observed (that is, a request must be picked up before it is delivered), and each customer must be served by exactly one route. We incorporate a fixed penalty associated with each additional customer after the first customer assigned to a route. This comes from the industry requirement to compensate truck drivers for the additional work. We refer to this penalty as the 'stop cost' of a route. We assume this value to be proportional to the number of additional customers assigned to a route. We assume each vehicle possesses the same capacity, and that vehicles cannot be empty for any portion of their route. Finally, we do not consider time in this problem. This stems from the notion that operations are on the order of days, and the requests refer to freight rather than people.

We seek to identify the set of routes R that partitions the set of customers, and minimizes the following objective:

$$\sum_{r \in R} F(s_r, t_r) d_r + c_r$$

where s_r, and t_r are the original and terminal nodes for route r, respectively, c_r is the aforementioned stop cost, and $F(x, y)$ is a given function of the original and terminal nodes of a route. For a route r that begins and ends in regions of high demand, $F(s_r, t_r)$ will be small.

Approach In our work, we have used Li and Lim's 2003 benchmark data set, designed for the Pickup and Delivery Problem with Time Windows, to create a sample model. The data set has instances for 50, 100, 200, 300, 400, and 500 customers. We utilize the subset of instances in this data set associated with longer planning horizons (to encourage indirect routes), and with locations that are either uniformly distributed or clustered. We have 10 instances for each problem size. We modify this data set for our problem by assigning demand levels to each node. We determine demand level of a node by the quantity of nearby nodes. Demand levels assume one of three values: high (1), medium (2), and low (3). Our assignment ensures roughly one quarter of the nodes are high demand, one quarter are low demand, and one half are medium demand. The value of F function from the objective for a particular route is determined by the product of the demand levels of the original and terminal nodes of the route.

Due to the complications caused by the $F(x, y)$ coefficient in the objective, we have focused on developing efficient heuristic approaches. We describe one of the more successful approaches: an iterated matching-based savings algorithm. The idea is that starting from some initial solution (say, the trivial solution, where each customer gets his/her own truck), construct a network, where each node is a route from the initial solution. Then, compute all of the edge weights, or some subset of the edge weights, in the complete network. Edge weights are determined by the savings associated with combining the two nodes (routes). Once this network is computed, solve the corresponding maximum weight matching problem. This well-studied subproblem can be solved in $O(n^3)$ time (e.g., Kolmogorov (2009)). This will result in a set of route matchings. We select all the matchings associated with positive savings, and combine the corresponding routes, producing a new solution. We can then iterate once again, and continue until some stopping criterion (in our work so far, we simply stop after k iterations). This approach is similar to that described in Desrochers and Verhoog (1989), as well as Wark and Holt (1994). This approach is F-function agnostic, in that it does not rely on any assumptions about the function rewarding/penalizing route endpoints.

There are two key steps in this algorithm: the determination of edge weights, and the actual matching algorithm. The latter is solved by a blossom algorithm. The former is solved by either dynamic programming or a random sampling approach, depending on the problem size. In the case of combining two short routes, identifying the optimal combination can be done relatively quickly by a dynamic program. However, when the routes become larger, we switch to sampling sequence-preserving combinations of the routes. We have also tried fully relying on the sampling approach.

Results We benchmark the effectiveness of our methods by measuring improvement upon the "trivial solution", i.e., the solution in which each customer gets his/her own truck. The following table presents our results on our data set. Each value is the average over 10 different instances (9 instances in the case of 500 customers):

Table 1 Performance of our heuristic.

Number of Customers	Improvement on Trivial Solution	Runtime (seconds)
50	76.31%	42.08
100	71.59%	144.61
200	72.44%	605.15
300	74.28%	1379.37
400	74.45%	2473.93
500	73.32%	4368.10

Values are averaged over 10 instances (9 instances for the 500 customer case).

The results are rather consistent across different problem sizes, with improvement on objective values usually within 70-75%. The run times quickly reach several minutes as the number of customers rises into the hundreds.

Conclusions We have presented a new kind of vehicle routing problem applicable in numerous real world settings. In particular, an objective function dependent on the origin location and destination location of the routes (via the F function) in addition to the distance traveled can be calibrated to match several settings in which preference for one location over another exists (e.g., nonuniform traffic/population density, adjacency to hospitals/refueling stations, etc.). The presence of such a function can dramatically change the character of the optimal solutions, and thus traditional mechanisms for solving vehicle routing problems (which usually seek to only minimize total distance) may not be appropriate. However, our heuristic approach is agnostic to F, and, thus, seems like a suitable solver for such problems. In future work, we may seek to fully exploit the specific nature of F.

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