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An Integrated Method to Solve the Healthcare Facility Layout Problem with Area Constraint

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Abstract In this paper, the healthcare facility layout problem (HFLP) with an area constraint is considered, to get a proper departments arrangement in a healthcare facility such that the operating cost and system efficiency are guaranteed. We propose an integrated method to consider both quantitative and qualitative criteria to get a synthesize rank of the feasible alternatives. On qualitative aspects, each layout alternative has attributes to be evaluated, the evaluation values given by experts are in the form of intuitionistic fuzzy sets. We assume that the weights of attributes and experts are not known or partially known in advance, and each expert has different weights on different attributes. An illustrative example is shown to demonstrate the application of the proposed methodology.

Keywords Healthcare Facility Layout Problem, Multi-attribute Group Decision Making, Experts Consistency, Mathematical Programming

1 Introduction

The healthcare facility layout problem (HFLP) is a new application of the classical facility layout problem (FLP), which attracts more and more attention from researchers and practitioners. As its importance role played in industrial world, a good arrangement of departments and work stations can reduce the operating cost and meanwhile, maximize the diagnosis efficiency. It is known

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that the transportation cost based on the flows and distances between departments encompasses 20% to 50% of the total operating cost, and can be reduced by, at least from 10% to 30%, by improving the layout design [22]. It has to be noted that only the cost reduction is far from enough, layout problem in the healthcare facility is more complicated because of its requirement on the timely treatment of emergency patients, the infection control and isolation, the services satisfaction etc. So many other factors, which could not be quantized in a mathematical model should also be considered.

In this paper, we present an integrated methodology to consider both quantitative and qualitative factors in solving the HFLP. We first formulate the HFLP as a special linear programming M-1, to generate m layout alternatives. In order to get enough diversity, we normally choose the one with the smallest transportation cost, then choose one every other until m is reached. Second we introduce a novel multi-attribute group decision making (MAGDM) method to get the qualitative rankings of m alternatives. We lastly synthesize the quantitative and qualitative rankings and output the final solution.

Suppose there is a rectangular healthcare facility, we use W to denote its width and H to denote its height. Let N be the number of rectangular departments. The position of department i ($i \in 1, 2, 3, ...N$) is denoted by the coordinates of its center (x_i, y_i), and the width and the height of department i are denoted by w_i and h_i, respectively. The HFLP model can be formulated as following:

(M-1)

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} f_{ij} c_{ij}$$

$$\text{s.t. } |y_i - y_j| \geq \frac{h_i + h_j}{2}, \text{ when } |x_i - x_j| < \frac{w_i + w_j}{2},\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

$$|x_i - x_j| \geq \frac{w_i + w_j}{2}, \text{ when } |y_i - y_j| < \frac{h_i + h_j}{2},\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

$$\frac{w_i}{2} \leq x_i \leq W - \frac{w_i}{2},\ i = 1, \ldots, N$$

$$\frac{h_i}{2} \leq y_i \leq H - \frac{h_i}{2},\ i = 1, \ldots, N$$

$$d_{ij} = |x_i - x_j| + |y_i - y_j|,\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

$$\varphi_i \times W \times H \leq w_i h_i \leq W \times H, \varphi_i \in (0, 1),\ i = 1, \ldots, N$$

$$0 < \sum_{i=1}^{N} \varphi_i \leq 1,\ i = 1, \ldots, N$$

where d_{ij} denotes the rectilinear distance between the centers of departments i and j, f_{ij} is the interdepartmental flows and c_{ij} is the transportation cost per unit distance between departments i and j. The objective function aims to minimize the total transportation cost among all departments. The first four constraints require that the departments cannot overlap each other and are entirely inside the facility. The dimensions of departments are unknown, area

proportional ratios φ_i are given in advance, to denote the specific constraint on department areas.

MAGDM method is generally used to evaluate the qualitative properties in decision theory. For alternatives with multiple attributes, a group of experts give their evaluation (in scores or orders) on each attribute. The method then finds a way to output the solution which reflects the experts' opinions as much as possible.

There are several points to note on this method. One is about the consensus of the experts' opinions. Several kinds of distance definitions were applied in literature [4,5,38] to indicate the difference between opinions, a smaller distance means the less difference. In this paper we introduce a new cosine angle method to reflect the consistency of two experts' opinions, the larger is the cosine value (close to 1), the higher consistency are the opinions. Following is a simple but clear example:

Suppose three experts are evaluating on given alternatives, the final scores are $\mathbf{v_1} = \{1, 2, 3\}$, $\mathbf{v_2} = \{2, 4, 6\}$, $\mathbf{v_3} = \{3, 2, 1\}$. Assume the bigger number means the higher preference, we can see that the expert 1 and expert 2 have the same opinion to choose the third alternative. If the distance criterion is applied, we can see that the distance between $\mathbf{v_1}$ and $\mathbf{v_2}$ is $\sqrt{(2-1)^2 + (4-2)^2 + (6-3)^2} \approx 3.74$, which is larger than that between $\mathbf{v_1}$ and $\mathbf{v_3}$ (2.83), but it is so clear that expert 1 and expert 3 have opposite opinions. Cosine angle value method is more reliable, by taking the evaluation values as vectors in space.

One more advantage of cosine angle method is also showed in the instance above, it reduces the influence of different scoring habits of the experts. Because we only care about the evaluation order of the alternatives, not the absolute evaluation value. The cosine angle value will be defined formally as the *experts consistency* in the later part.

Another issue with applying the group decision method is about the weight of the experts. Lots of papers considered this set of problems, with experts' weight known or partially known [16,27]. But they all take the weight as a scalar, rather than a vector, which means all experts are labeled with same weight on various attributes. In this paper we assume that the experts may have different specialties, their weights should be hence defined corresponding to each attribute.

The rest of this paper is organized as follows. Section 2 presents the related works as literatures review. Section 3 describes the definitions and operators used in MAGDM. Section 4 introduces our integrated method. An illustrative example is given in Section 5. In Section 6 we give the conclusion and talk about the future works.

2 Literature review

For the last a few decades, FLP has attracted the attention of researchers and practitioners because of its practical utility and interdisciplinary importance.

Montreuil [19], Sherali et al. [21], Castillo et al. [6] and Konak et al. [17] presented different modeling and solution techniques for layout planning problems in general. The application of FLP in healthcare facility was first introduced by Elshafei in 1977 [10]. He modeled a hospital layout problem as a quadratic assignment problem (QAP) and developed a heuristic procedure to solve it. Arnolds et al. [1] presented a robust hospital layout plan by using the discrete event simulation(DES), with the assumption of stochastic patient flows. The DES model can easily be adapted to different layout plans by changing the location assignment for the departments. Chraibi et al. [9] proposed a mixed integer linear programming model on the operating theatre layout problem. The authors aims to minimize the movement costs generated by patients, doctors, medical and non-medical staffs, and meanwhile to maximize the desired closeness among activities. Assem et al. [2] generated operating theatre layout design based on the graph theoretic approach, a layout score was calculated for each design. Lin et al. [18] proposed a systematic layout planning method and they applied fuzzy constraint theory to evaluate the layout schemes. Abdelahad et al. [8] proposed a particle swarm optimization algorithm to solve the operating theater layout problem. The algorithm employs a constructive heuristic to generate initial feasible solutions, and find the final solution in polynomial time. It has to be note that Zhong and Tang [41] has encouraged more researchers to focus on the Healthcare problem. For the scheduling problem occurs in hospital, [11, 40, 24] have done lots of work with good algorithms. Because of the complicated interactions between the users of a healthcare resource, there are also papers considering the problem from the point of view of game theory [13, 12].

Owing to the uncertainty and vagueness of the objects and the ambiguity of human thinking, it is difficult for decision makers to express their preferences over alternatives by precise values. The fuzzy set theory initiated by Zadeh [36] is a powerful tool to characterize such uncertainty and fuzziness, which has been extensively investigated by many researchers from different disciplines. In the field of MAGDM, interval numbers [37], triangular fuzzy numbers [25] and trapezoidal fuzzy numbers [23] are usually used to represent the alternative ratings on attributes. On the other hand, because the experts in practical MAGDM problems usually come from various research areas and may have many differences in knowledge structure, express abilities, evaluation levels etc., each expert should be assigned a weight that reflects the corresponding importance in the whole evaluating group. Many useful and valuable methods have been proposed to determine the weights of experts over the last decades. Xu [30] utilized some deviation measures between additive linguistic preference relations to give some straightforward formulas for determining the experts weights. The last but the most important phase in the MAGDM process is how to take advantage of decision information to sort the alternatives and to select the best one(s). Many classical ranking methods, such as the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [15], the VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method [20] have been proposed.

3 Definitions and Operators

Considering the experts' evaluation score on alternatives, plenty of expressions other than real number were used for recent decades, including linguistic [26], triangular fuzzy [25], interval-valued intuitionistic fuzzy [34] etc. In this paper we use the intuitionistic fuzzy set(IFS) introduced by Atanassov [3] in 1986.

3.1 Definition and operation of Intuitionistic fuzzy sets

Definition 1 Let a set X be a universe of discourse. An IFS A is an object which has the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{1}$$

where $\mu_A(x)$: X $\rightarrow$ [0, 1] and $\nu_A(x) : X \rightarrow [0, 1]$ are membership function and non-membership function, respectively, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x to A, for all $x \in X$. In addition, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation degree of $x \in A$, representing the degree of hesitancy of x to A. It is obvious that $0 \leq \pi_A(x) \leq 1, \forall x \in X$. If the $\pi_A(x)$ is small, then the value of x is easier to determine; while if $\pi_A(x)$ is large, then x is more difficult to determine.

For an IFS, the pair $(\mu_A(x), \nu_A(x))$ is called an intuitionistic fuzzy number (IFN) and each IFN can be simply denoted as $\alpha = (\mu_\alpha, \nu_\alpha)$, where $\mu_\alpha \in [0, 1]$, $\nu_\alpha \in [0, 1]$ and $\mu_\alpha + \nu_\alpha \leq 1$. For an IFN $\alpha = (\mu_\alpha, \nu_\alpha)$, if the value μ_α gets bigger and the value ν_α gets smaller, then the IFN $\alpha = (\mu_\alpha, \nu_\alpha)$ gets larger. Obviously, $\alpha^+ = (1, 0)$, $\alpha^- = (0, 1)$ are the largest and the smallest IFNs, respectively. In addition, $\mathrm{S}(\alpha) = \mu_\alpha - \nu_\alpha$ and $\mathrm{H}(\alpha) = \mu_\alpha + \nu_\alpha$ are called the score and accuracy degree of α, respectively.

For any three IFNs $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1}), \alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ and $\alpha = (\mu_\alpha, \nu_\alpha)$, the following operational laws were introduced by Xu and Yager [31], Xu [28]:
(1) $\alpha_1 + \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2})$,
(2) $\alpha_1 \times \alpha_2 = (\mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1} + \nu_{\alpha_2} - \nu_{\alpha_1}\nu_{\alpha_2})$,
(3) $\lambda\alpha = (1 - (1 - \mu_\alpha)^\lambda, \nu_\alpha^\lambda), \lambda > 0$,
(4) $\alpha^\lambda = (\mu_\alpha^\lambda, 1 - (1 - \nu_\alpha)^\lambda), \lambda > 0$.

To compare any two IFNs α_1 and α_2, the following method was proposed by Xu and Yager [28] based on the score and accuracy functions:
(1) If $\mathrm{S}(\alpha_1) < \mathrm{S}(\alpha_2)$, then $\alpha_1 < \alpha_2$;
(2) If $\mathrm{S}(\alpha_1) = \mathrm{S}(\alpha_2)$, and

(a) If $\mathrm{H}(\alpha_1) < \mathrm{H}(\alpha_2)$, then $\alpha_1 < \alpha_2$;
(b) If $\mathrm{H}(\alpha_1) = \mathrm{H}(\alpha_2)$, then $\alpha_1 = \alpha_2$.

To measure the deviation between any two IFNs, Xu and Yager [32] defined the following distance:

Definition 2 Let $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ be two IFNs, then the distance between α_1 and α_2 is defined as

$$D(\alpha_1, \alpha_2) = |\alpha_1 - \alpha_2| = \frac{1}{2}(|\mu_{\alpha_1} - \mu_{\alpha_2}| + |\nu_{\alpha_1} - \nu_{\alpha_2}| + |\pi_{\alpha_1} - \pi_{\alpha_2}|) \quad (2)$$

3.2 The IFWA and IFA operator

Following the conceptions and operations of IFNs, let Ω be the set of all IFNs, Xu [28] introduced the intuitionistic fuzzy weighted averaging (IFWA) operator as follows:

Definition 3 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$, $i = 1, 2, ..., n$ be a collection of IFNs and let IFWA: $\Omega^n \to \Omega$, if

$$\begin{aligned} IFWA(\alpha_1, \alpha_2, ..., \alpha_n) &= \sum_{i=1}^{n} \omega_i \alpha_i \\ &= \Big(1 - \prod_{i=1}^{n}(1 - \mu_{\alpha_i})^{\omega_i}, \prod_{i=1}^{n}(\nu_{\alpha_i})^{\omega_i}\Big), i = 1, 2, ..., n \end{aligned} \quad (3)$$

then IFWA is called an intuitionistic fuzzy weighted averaging operator, where $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weight vector of α_i(i = 1,2,...,n), with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$. And if $\boldsymbol{\omega} = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$, then the IFWA operator is reduced to the intuitionistic fuzzy averaging (IFA) operator:

$$\begin{aligned} IFA(\alpha_1, \alpha_2, ..., \alpha_n) &= \frac{1}{n}\sum_{i=1}^{n} \alpha_i \\ &= \Big(1 - \prod_{i=1}^{n}(1 - \mu_{\alpha_i})^{\frac{1}{n}}, \prod_{i=1}^{n}(\nu_{\alpha_i})^{\frac{1}{n}}\Big), i = 1, 2, ..., n \end{aligned} \quad (4)$$

4 The integrated HFLP approach

Our integrated method includes two parts. In the first part we formulate the healthcare facility layout problem as a special linear programming $M - 1$ and solve it for at least $2m$ times, to get m good enough layout alternatives respecting to their cost and diversity. The alternatives are denoted as $A = \{A_1, A_2, \ldots, A_m\}$, and let $X = \{x_1, x_2, \ldots, x_n\}$ be the n attributes of alternative, $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of attributes, $0 \le \omega_j \le 1, \sum_{j=1}^{n} \omega_j = 1$. In this paper, we assume that the weights of attributes are completely unknown, which is very natural in reality because of the difficulties to balance. Certainly our method can solve the case with known or partially known weight. Let $E = \{e_1, e_2, \ldots, e_p\}$ be a group of p experts. They give their evaluations on each attribute of each alternative, which generate their

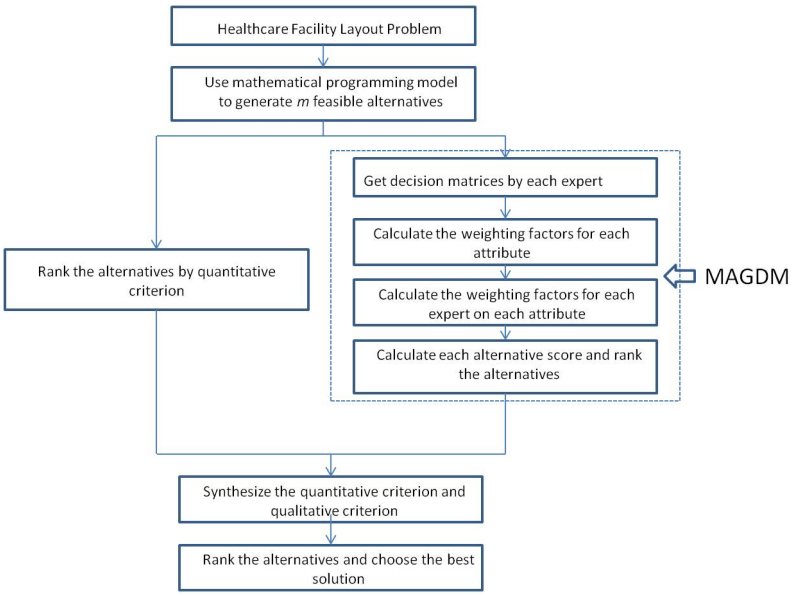


Fig. 1: Flowchart of the integrated FHLP approach

decision matrices. The evaluation scores take the form of intuitionistic fuzzy set. Similarly the weighting vectors of the experts on attributes are not known. Our method find a way to get the weighting factors and use the technique for order preference by similarity to ideal solution (TOPSIS) method, proposed by Hwang and Yoon [15], to calculate the alternative score. We lastly synthesize the quantitative and qualitative ranking and output the final solution.

The flowchart in Fig 1 shows the procedures of proposed integrated method.

Step 1 Use mathematical programming model to generate m feasible alternatives.

In order to get enough diversity, we normally choose the one with the smallest transportation cost, then choose one every other until m is reached.

Step 2 Get the decision matrices of each expert.

Let $\mathbf{R^k} = (\alpha_{ij}{}^k)_{m\times n}$ be an intuitionistic fuzzy decision matrix provided by the expert $e_k(k \in p)$. Therefore, the MAGDM problem with IFNs can be represented as the following matrix form:

$$\begin{aligned}\mathbf{R^k} &= (\alpha_{ij}{}^k)_{m\times n} \\ &= \begin{pmatrix} (\mu^k_{\alpha_{11}}, \nu^k_{\alpha_{11}}) & (\mu^k_{\alpha_{12}}, \nu^k_{\alpha_{12}}) & \dots & (\mu^k_{\alpha_{1n}}, \nu^k_{\alpha_{1n}}) \\ (\mu^k_{\alpha_{21}}, \nu^k_{\alpha_{21}}) & (\mu^k_{\alpha_{22}}, \nu^k_{\alpha_{22}}) & \dots & (\mu^k_{\alpha_{2n}}, \nu^k_{\alpha_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu^k_{\alpha_{m1}}, \nu^k_{\alpha_{m1}}) & (\mu^k_{\alpha_{m2}}, \nu^k_{\alpha_{m2}}) & \dots & (\mu^k_{\alpha_{mn}}, \nu^k_{\alpha_{mn}}) \end{pmatrix} (k \in p)\end{aligned} \tag{5}$$

Step 3 Calculate the weighting factors of each attribute.

Clearly, in the practical MAGDM process the opinion of the group should be consistent with the individual experts opinion to the greatest extent. Assume that the common will of the group is the neutrality decision of group, for convenience of description, which is presumed to be provided by the neutrality expert e^*. Similar to Yue [35], it is reasonable to assume that the neutrality decision of group is the mean of all individual experts decision. Thus the neutrality decision matrix denoted by R^* can be determined by using the following formula as:

$$\begin{aligned}\mathbf{R}^* &= (\alpha_{ij}{}^*)_{m\times n} \\ &= \begin{pmatrix} (\mu^*_{\alpha_{11}}, \nu^*_{\alpha_{11}}) & (\mu^*_{\alpha_{12}}, \nu^*_{\alpha_{12}}) & \dots & (\mu^*_{\alpha_{1n}}, \nu^*_{\alpha_{1n}}) \\ (\mu^*_{\alpha_{21}}, \nu^*_{\alpha_{21}}) & (\mu^*_{\alpha_{22}}, \nu^*_{\alpha_{22}}) & \dots & (\mu^*_{\alpha_{2n}}, \nu^*_{\alpha_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu^*_{\alpha_{m1}}, \nu^*_{\alpha_{m1}}) & (\mu^*_{\alpha_{m2}}, \nu^*_{\alpha_{m2}}) & \dots & (\mu^*_{\alpha_{mn}}, \nu^*_{\alpha_{mn}}) \end{pmatrix} \end{aligned} \tag{6}$$

using Eq. 4 we obtain $\mu^*_{\alpha_{ij}} = 1 - \prod_{k=1}^{p}(1-\mu^k_{\alpha_{ij}})^{\frac{1}{p}}$, $\nu^*_{\alpha_{ij}} = \prod_{k=1}^{p}(\nu^k_{\alpha_{ij}})^{\frac{1}{p}} (i \in m, j \in n)$.

In the real life, there always exist some differences between the ideal point of attribute values and the vector of attribute $x_j (j = 1, 2, ..., n)$ values corresponding to the alternative $A_i (i = 1, 2, ..., m)$. By **Definition** 2, in what follows Xu and Da [29] define the distance d_i between the overall value α^+_{ij} of ideal point and the overall value α_{ij} of the alternative A_i:

$$d_i = \sum_{j=1}^{n} (D(\alpha^+_{ij}, \alpha_{ij})\omega_j)^2, i = 1, 2, ..., m \tag{7}$$

Obviously, the smaller is d_i, the better is the alternative A_i. Thus, a reasonable weight vector $\boldsymbol{\omega}^* = (\omega^*_1, \omega^*_2, ..., \omega^*_n)^T$ should be determined such that all the distances $d_i (i = 1, 2, ..., m)$ are as smaller as possible, which means we have to minimize the following distance vector: $d(\omega) = (d_1, d_2, ..., d_m)$. In order to do this, Xu and Da [29] established the following multiple objective optimization model:

(M-2)

$$\min F(\omega) = (d_1, d_2, ..., d_m)$$

$$\text{s.t. } \textstyle\sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0$$

Generally, all the objectives are fairly competitive and there is no preference relationship among them, therefore the above model can be transformed into the following goal programming problem:

(M-3)

$$\min F(\omega) = \textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} (D(\alpha^+_{ij}, \alpha_{ij})\omega_j)^2$$

$$\text{s.t. } \textstyle\sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0$$

To solve this model, a Lagrange function is constructed, and we can get:

$$\omega_j^* = \frac{1}{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{m} D^2(\alpha_{ij}^+, \alpha_{ij})}} / \sum_{i=1}^{m} D^2(\alpha_{ij}^+, \alpha_{ij}) \tag{8}$$

which can be used as the weight vector of attributes. Obviously, $\omega_j \geq 0$, for all j.

Step 4 Calculate the weighting factors of each expert on each attribute.

When the weight of experts are unknown or uncertain, generally we will try to decide it such that the whole consistency is maximized, which means minimize the influence of any opinion of a single expert. We define the experts consistency(EC) as below:

$$EC_k^*(x_j) = \frac{\mathbf{a_k} \cdot \mathbf{a_*}}{|\mathbf{a_k}||\mathbf{a_*}|} = \frac{\sum_{i=1}^{m} S(\alpha_{ij}^k) S(\alpha_{ij}^*)}{\sqrt{\sum_{i=1}^{m} S^2(\alpha_{ij}^k)} \sqrt{\sum_{i=1}^{m} S^2(\alpha_{ij}^*)}} \tag{9}$$

where the evaluation of expert e^k for attribute x_i of all alternatives is the column vector $\mathbf{a_k} = (S(\alpha_{1j}^k), S(\alpha_{2j}^k, ..., S(\alpha_{mj}^k)))$, and that of the expert e^* is the column vector $\mathbf{a_*} = (S(\alpha_{1j}^*), S(\alpha_{2j}^*), ..., S(\alpha_{mj}^*))$.

There are some real-life situations that the information about the weights of the experts is not completely unknown but partially known. For these cases, let Γ be the set of information known of weights, (1) a weak ranking: $\Gamma_1 = \{\lambda_i \geq \lambda_j\}$; (2) a strict ranking: $\Gamma_2 = \{\lambda_i - \lambda_j \geq \beta_i, (\beta_i \geq 0)\}$; (3) a ranking of differences: $\Gamma_3 = \{\lambda_i - \lambda_j \geq \lambda_k - \lambda_l \, (i \neq j \neq k \neq l)\}$; (4)a ranking with multiples: $\Gamma_4 = \{\lambda_i \geq \beta_i \lambda_j, (0 \leq \beta_i \leq 1)\}$; (5)an interval form: $\Gamma_5 = \{\beta_i \leq \lambda_i \leq \beta_i + \epsilon, (0 \leq \beta_i \leq \beta_i + \epsilon \leq 1)\}$. The structure forms of the weights of the experts usually consist of several sets of the above basic sets or may contain all the five basic sets, which depend on the characteristic and need of the real-life decision problems. $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5$, consequently, we have to choose the experts weight matrix $\mathbf{Q}$ to maximize overall consensus between all the experts and the ideal expert e^* for all the alternatives with all the attributes from the perspective of the ranking of decision information. To this end, we construct a non-linear programming model to determine the experts weights as below:

(M-4)

$$\max EC(\lambda_{kj}) = \sum_{k=1}^{p} \sum_{j=1}^{n} (EC_k^*(x_j) \lambda_{kj} \omega_j)$$

$$\text{s.t. } \lambda_{kj} \in \Gamma, \sum_{k=1}^{p} \lambda_{kj} = 1, \lambda_{kj} > 0$$

In the MAGDM process, after obtaining the optimal weight matrix $\mathbf{Q}$ of experts, we usually need to aggregate all individual decision matrices $\mathbf{R^k}(k \in$

p) into a collective decision $\mathbf{R}$ by using the following formula:

$$\begin{aligned}\mathbf{R} &= (\alpha_{ij})_{m\times n} \\ &= \begin{pmatrix} (\mu_{\alpha_{11}}, \nu_{\alpha_{11}}) & (\mu_{\alpha_{12}}, \nu_{\alpha_{12}}) & \dots & (\mu_{\alpha_{1n}}, \nu_{\alpha_{1n}}) \\ (\mu_{\alpha_{21}}, \nu_{\alpha_{21}}) & (\mu_{\alpha_{22}}, \nu_{\alpha_{22}}) & \dots & (\mu_{\alpha_{2n}}, \nu_{\alpha_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{\alpha_{m1}}, \nu_{\alpha_{m1}}) & (\mu_{\alpha_{m2}}, \nu_{\alpha_{m2}}) & \dots & (\mu_{\alpha_{mn}}, \nu_{\alpha_{mn}}) \end{pmatrix}\end{aligned} \tag{10}$$

using Eq. 3 we obtain $\mu_{\alpha_{ij}} = 1 - \prod_{k=1}^{p}(1-\mu_{\alpha_{ij}}^{k})^{\lambda_{kj}}$, $\nu_{\alpha_{ij}} = \prod_{k=1}^{p}(\nu_{\alpha_{ij}}^{k})^{\lambda_{kj}} (i \in m, j \in n)$.

Step 5 Calculate each alternative score and rank the alternatives.

We use TOPSIS method to calculate alternative score. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution (Hwang and Yoon [15]; Chen et al. [7]). Suppose that a multi-attribute decision making problem has m alternatives A_i $(i = 1, 2, ..., m)$ and n decision attributes X_j $(j = 1, 2, ..., n)$. Each alternative is assessed with respect to the n criteria. All the performance ratings assigned to the alternatives with respect to each criterion form a decision matrix denoted by $(\alpha_{ij})_{m\times n}$. Then, the TOPSIS method can be summarized as the following steps:

TOPSIS-Step 1. Construct the normalized decision matrix $\mathbf{R} = (\alpha_{ij})_{m\times n}$.
TOPSIS-Step 2. Determine the intuitionistic fuzzy positive ideal solution(IFPIS) and intuitionistic fuzzy negative ideal solutions(IFNIS) as following:

$$\alpha^{+} = \{\alpha_{i1}^{+}, \alpha_{i2}^{+}, ..., \alpha_{in}^{+}\} \tag{11}$$

where $\alpha_{ij}^{+} = (\max\{\mu_{1j}, \mu_{2j}, ..., \mu_{mn}\}, \min\{\nu_{1j}, \nu_{2j}, ..., \nu_{mn}\}), j = 1, 2, ...n$

$$\alpha^{-} = \{\alpha_{i1}^{-}, \alpha_{i2}^{-}, ..., \alpha_{in}^{-}\} \tag{12}$$

where $\alpha_{ij}^{-} = (\min\{\mu_{1j}, \mu_{2j}, ..., \mu_{mn}\}, \max\{\nu_{1j}, \nu_{2j}, ..., \nu_{mn}\}), j = 1, 2, ...n$

TOPSIS-Step 3. Calculate the Euclidean distances of each alternative from the positive ideal solution and the negative ideal solution as following:

$$D_i^{+} = \sum_{j=1}^{n} \omega_j D(\alpha^{+}, \alpha_{ij}) \tag{13}$$

$$D_i^{-} = \sum_{j=1}^{n} \omega_j D(\alpha^{-}, \alpha_{ij}) \tag{14}$$

TOPSIS-Step 4. Calculate the relative closeness of each alternative to the ideal solution as following:

$$RC_i^{+} = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \tag{15}$$

where RC_i^+ is the relative closeness of the alternative A_i concerning to α_{ij}^+.

TOPSIS-Step 5. Rank the alternatives by alterative scores.

Step 6 Determine the final solution by synthesizing transportation costs and alternative scores.

We use the following equations to normalize the quantitative criteria and qualitative criteria.

Firstly, transform the performance measures of cost-type criteria using the formula as following:

$$c_i = \frac{max\alpha_i - \alpha_i}{max\alpha_i - min\alpha_i} \tag{16}$$

where α_i donates cost-type criteria value.

Then, transform the performance the measures of benefit-type criteria using the formula as following:

$$b_i = \frac{\beta_i - min\beta_i}{max\beta_i - min\beta_i} \tag{17}$$

where β_i donates benefit-type criteria value.

In order to synthesize the transportation costs and the alternative score, we use ρ_1, ρ_2 to denote their weights, which are given in advance as proposed in [8], satisfying that $0 \leq \rho_1 \leq 1, 0 \leq \rho_2 \leq 1, \sum_{j=1}^{2} \rho_j = 1$.

5 Illustrative example

In this section, to show the procedure and the applicability of our proposed methodology, a simple example is presented. First, **Table** 1 shows the details with the names and the lower bounds of department areas. We assume "$TotalArea = Width \times Height = 1000 \times 800m^2$. Then we utilize **M-1** to de-

Table 1: The departments name and their least area.

Department No	Department Name	Minimum Area(m2)
1	Image Diagnosis Rooms	1/10×TotalArea
2	Laboratory Diagnosis Rooms	1/6×TotalArea
3	Surgery Rooms	1/6×TotalArea
4	Wards	1/4×TotalArea

termine the departments' position and area. The flows and unit-cost between each department are showed as following:

$$\textbf{flow} = \begin{bmatrix} 20 & 30 & 10 & 30 \\ 20 & 40 & 10 & 30 \\ 10 & 30 & 30 & 30 \\ 10 & 30 & 60 & 10 \end{bmatrix}, \textbf{unit-cost} = \begin{bmatrix} 10 & 30 & 10 & 30 \\ 20 & 10 & 10 & 30 \\ 20 & 30 & 10 & 30 \\ 20 & 30 & 50 & 10 \end{bmatrix}.$$

Table 2: The transportation cost of each alternative.

FLAs	TC
A1	3942286
A2	4164263
A3	5254432
A4	6410997
A5	7264239

Table 3: The position and area of A1.

Department No	Position(x,y)	Area(w,h)
1	(500.0000, 40.00000)	(1000.000, 80.00000)
2	(857.1429, 446.6667)	(285.7143, 466.6667)
3	(500.0000, 146.6667)	(1000.000, 133.3333)
4	(500.0000, 446.6667)	(428.5714, 466.6667)

Table 4: The position and area of A2.

Department No	Position(x,y)	Area(w,h)
1	(393.9433, 660.5028)	(286.7442, 278.9943)
2	(393.9433, 288.5104)	(286.7442, 464.9906)
3	(620.6487, 400.0000)	(166.6667, 800.0000)
4	(125.5712, 400.0000)	(250.0000, 800.0000)

Table 5: The position and area of A3.

Department No	Position(x,y)	Area(w,h)
1	(104.4730, 191.4370)	(208.9460, 382.8740)
2	(728.8650, 577.0596)	(343.3143, 388.3711)
3	(383.0770, 574.3008)	(348.2619, 382.8536)
4	(472.2976, 193.0138)	(526.7032, 379.7205)

The model can be easily solved using the LINGO 11.0 software. The facility layout alternatives(FLAs) and the transportation cost(TC) are show in **Table** 2. The department's position and area are show in **Table** 3 **-** 7.

In order to have an intuitive impression, we use "CorelDRAW" software to draw the each facility layout alternative, show in **Figure** 2 **-** 6.

The assessments of five possible alternatives $A_i(i = 1, 2, 3, 4, 5)$ with respect to the attributes $x_j(j = 1, 2, 3, 4)$ provided by the experts $e_k(k = 1, 2, 3, 4)$, are expressed by the IFNs, listed in **Table** 8 **-** 11.

Table 6: The position and area of A4.

Department No	Position(x,y)	Area(w,h)
1	(55.67730, 359.2128)	(111.3546, 718.4256)
2	(532.5345, 421.5335)	(176.1495, 756.9329)
3	(277.9072, 200.1370)	(333.1051, 400.2740)
4	(810.3046, 536.4195)	(379.3908, 527.1609)

Table 7: The position and area of A5.

Department No	Position(x,y)	Area(w,h)
1	(281.4890, 291.2406)	(137.3435, 582.4812)
2	(817.4258, 617.4258)	(365.1484, 365.1484)
3	(106.4086, 486.7421)	(212.8172, 626.5157)
4	(675.0804, 153.8842)	(649.8393, 307.7684)

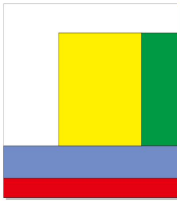


Fig. 2: A1

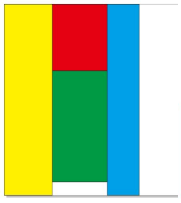


Fig. 3: A2

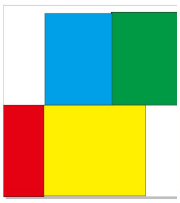


Fig. 4: A3

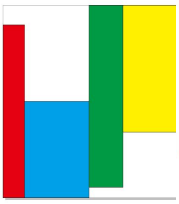


Fig. 5: A4



Fig. 6: A5

Table 8: Intuitionistic fuzzy decision matrix **R1**.

FLAs	x1	x2	x3	x4
A1	(0.5,0.2)	(0.3,0.4)	(0.4,0.3)	(0.3,0.4)
A2	(0.3,0.4)	(0.1,0.2)	(0.7,0.1)	(0.1,0.7)
A3	(0.6,0.2)	(0.3,0.4)	(0.5,0.1)	(0.1,0.5)
A4	(0.5,0.1)	(0.2,0.5)	(0.4,0.2)	(0.2,0.4)
A5	(0.1,0.3)	(0.7,0.1)	(0.5,0.2)	(0.2,0.5)

Table 9: Intuitionistic fuzzy decision matrix **R2**.

FLAs	x1	x2	x3	x4
A1	(0.4,0.2)	(0.3,0.4)	(0.4,0.3)	(0.3,0.4)
A2	(0.3,0.4)	(0.1,0.3)	(0.6,0.1)	(0.1,0.6)
A3	(0.6,0.1)	(0.3,0.4)	(0.7,0.1)	(0.1,0.7)
A4	(0.5,0.1)	(0.2,0.6)	(0.4,0.3)	(0.3,0.4)
A5	(0.1,0.3)	(0.6,0.1)	(0.5,0.2)	(0.2,0.5)

In the following, we employ the proposed method to solve the above selection problem. By Step 1, we utilize Eq. 8 to determine the optimal weights of the attributes. Before that, we first calculate the ideal decision matrix $\mathbf{R}^*$ using Eq. 6. The results are listed in **Table** 12. Utilize Eq. 11, we can get the ideal solution of the attribute values as following: $\alpha^+ =$

Table 10: Intuitionistic fuzzy decision matrix **R3**.

FLAs	x1	x2	x3	x4
A1	(0.4,0.1)	(0.4,0.2)	(0.2,0.3)	(0.3,0.2)
A2	(0.3,0.3)	(0.2,0.4)	(0.6,0.1)	(0.1,0.6)
A3	(0.6,0.1)	(0.4,0.3)	(0.5,0.1)	(0.1,0.5)
A4	(0.5,0.1)	(0.1,0.7)	(0.5,0.2)	(0.2,0.5)
A5	(0.3,0.4)	(0.6,0.2)	(0.6,0.1)	(0.1,0.6)

Table 11: Intuitionistic fuzzy decision matrix **R4**.

FLAs	x1	x2	x3	x4
A1	(0.6,0.2)	(0.4,0.4)	(0.4,0.3)	(0.3,0.4)
A2	(0.3,0.3)	(0.1,0.2)	(0.6,0.1)	(0.1,0.6)
A3	(0.7,0.1)	(0.3,0.5)	(0.5,0.1)	(0.1,0.5)
A4	(0.5,0.1)	(0.2,0.4)	(0.4,0.2)	(0.2,0.4)
A5	(0.1,0.5)	(0.6,0.1)	(0.5,0.3)	(0.3,0.5)

Table 12: Intuitionistic fuzzy decision matrix $\mathbf{R}^*$.

FLAs	x1	x2	x3	x4
A1	(0.4820,0.1682)	(0.3519,0.3364)	(0.3553,0.3000)	(0.3000,0.3364)
A2	(0.3000,0.3464)	(0.1261,0.6232)	(0.6278,0.1000)	(0.1000,0.6236)
A3	(0.6278,0.1189)	(0.3265,0.3936)	(0.5599,0.1000)	(0.1000,0.5439)
A4	(0.5000,0.1000)	(0.1761,0.5384)	(0.4267,0.2213)	(0.2263,0.4229)
A5	(0.1548,0.3663)	(0.6278,0.1189)	(0.5271,0.1861)	(0.2031,0.5233)

Table 13: EC between the individual experts opinions and the ideal experts opinion.

EC_k^*	x1	x2	x3	x4
EC_1^*	0.99046	0.97135	0.99047	0.99105
EC_2^*	0.98673	0.97948	0.97557	0.97406
EC_3^*	0.98743	0.88527	0.95841	0.95880
EC_4^*	0.98105	0.94392	0.98576	0.98631

[(0.6278, 0.1000),(0.6278, 0.1189),(06278, 0.1000),(0.3000, 0.3364)], then we have: $\boldsymbol{\omega^*} = (0.15057, 0.08926, 0.42944, 0.33073)^T$.

By Step 2, We utilize **M-4** to determine the optimal weights of the experts in each attribute. Before that, we first calculate the EC using Eq. 9. The results are listed in **Table** 13. Then, using model **M-4**, we establish the following objective programming model:

$\max \mathrm{EC}(\lambda_{kj}) = \sum_{k=1}^{p}\sum_{j=1}^{n} GC_k^*(x_j) \times \lambda_{kj} \times \omega_j$

$= 0.99046 \times \lambda_{11} \times 0.15057 + 0.97135 \times \lambda_{12} \times 0.08926 + 0.99047 \times \lambda_{13} \times 0.42944 +$
$0.99105 \times \lambda_{14} \times 0.33073 + 0.98673 \times \lambda_{21} \times 0.15057 + 0.97948 \times \lambda_{22} \times 0.08926 +$
$0.97557 \times \lambda_{23} \times 0.42944 + 0.97406 \times \lambda_{24} \times 0.33073 + 0.98743 \times \lambda_{31} \times 0.15057 +$
$0.88527 \times \lambda_{32} \times 0.08926 + 0.95841 \times \lambda_{33} \times 0.42944 + 0.95880 \times \lambda_{34} \times 0.33073 +$
$0.98105 \times \lambda_{41} \times 0.15057 + 0.94392 \times \lambda_{42} \times 0.08926 + 0.98576 \times \lambda_{43} \times 0.42944 +$
$0.98631 \times \lambda_{44} \times 0.33073$

s.t. $\lambda_{11} + \lambda_{21} + \lambda_{31} + \lambda_{41} = 1, \lambda_{11} \geq 0.3, \lambda_{21} \geq 0.1, \lambda_{31} \geq 0.1, \lambda_{41} \geq 0.1$
$\lambda_{12} + \lambda_{22} + \lambda_{32} + \lambda_{42} = 1, \lambda_{12} \geq 0.1, \lambda_{22} \geq 0.1, \lambda_{32} \geq 0.1, \lambda_{42} \geq 0.3$
$\lambda_{13} + \lambda_{23} + \lambda_{33} + \lambda_{43} = 1, \lambda_{13} \geq 0.1, \lambda_{23} \geq 0.3, \lambda_{33} \geq 0.1, \lambda_{43} \geq 0.1$
$\lambda_{14} + \lambda_{24} + \lambda_{34} + \lambda_{44} = 1, \lambda_{14} \geq 0.1, \lambda_{24} \geq 0.1, \lambda_{34} \geq 0.3, \lambda_{44} \geq 0.1$

By solving the above model, we can obtain the optimal weight matrix of the

Table 14: Intuitionistic fuzzy collective decision matrix **R**.

FLAs	x1	x2	x3	x4
A1	(0.4545,0.1414)	(0.3419,0.3732)	(0.3825,0.3000)	(0.3000,0.3249)
A2	(0.3000,0.3365)	(0.1105,0.2625)	(0.6536,0.1000)	(0.1000,0.6481)
A3	(0.6113,0.1231)	(0.3107,0.4156)	(0.5710,0.1000)	(0.1000,0.5171)
A4	(0.5000,0.1000)	(0.1905,0.5298)	(0.4108,0.2259)	(0.2106,0.4277)
A5	(0.2063,0.3646)	(0.6113,0.1072)	(0.5110,0.1943)	(0.1822,0.5281)

experts as below:

$$\mathbf{Q} = \begin{bmatrix} 0.3 & 0.1 & 0.5 & 0.5 \\ 0.1 & 0.5 & 0.3 & 0.1 \\ 0.5 & 0.1 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.1 & 0.1 \end{bmatrix}$$

By Step 3: we employ Eq. 10 to aggregate all individual decision matrices $\mathbf{R^k}$(k = 1,2,3,4) into a collective decision matrix **R**. The results are shown in **Table** 14.

By Step 4 : we utilize Eq. 11 and 12 to determine the IFPIS α^+ and the IFNIS α^-, which are obtained as following:
α^+ = [(0.6113,0.1000),(0.6113,0.1072),(0.6536,0.1000),(0.3000,0.4277)]
α^- = [(0.2063,0.3646),(0.1105,0.5298),(0.3825,0.3000),(0.1000,0.6481)]

By Step5 : we utilize Eq. 13 and 14 to calculate the separation measures D_i^+ and D_i^- of each alternative $A_i(i = 1, 2, 3, 4, 5)$ from the IFPIS α^+ and the IFNIS α^-, respectively. The results obtained are shown as following:
$D_1^+ = 0.31116$ $D_2^+ = 0.26103$ $D_3^+ = 0.17965$ $D_4^+ = 0.24963$ $D_5^+ = 0.25099$
$D_1^- = 0.25479$ $D_2^- = 0.24449$ $D_3^- = 0.31175$ $D_4^- = 0.22084$ $D_5^- = 0.21245$

By Step 6 : we employ Eq. 15 to calculate the indices of the alternative A_i(i = 1,2,3,4,5) as following:
$RC_1 = 0.4502$ $RC_2 = 0.4836$ $RC_3 = 0.6433$ $RC_4 = 0.4694$ $RC_5 = 0.4585$

By Step 7 : we finally rank the alternatives $A_i(i = 1, 2, 3, 4, 5)$. As mentioned previously, Obviously, we just need to rank $RC_i(i = 1, 2, 3, 4, 5)$ in descending order : $RC_3 > RC_2 > RC_4 > RC_5 > RC_1$.

In summary, we have the quantitative criteria and qualitative criteria of each alternative, show in **Table** 15. Transform the performance measures of TC using Eq. 16 , RC using Eq. 17. In our example, the weights of TC and RC are provided by hospital managers. The value is 0.7 and 0.3. The rank result is show in **Table** 16.

6 Conclusions and future works

In this paper, we present an integrated method to solve the healthcare facility layout problem, by considering both quantitative criteria and qualitative

Table 15: Quantitative criteria and qualitative criteria RC from each alternative.

FLAs	TC	RC
A1	3942286	0.4502
A2	4164263	0.4836
A3	5254432	0.6433
A4	6410997	0.4694
A5	7264239	0.4585
Max	7264239	0.6433
Min	3942286	0.4502

Table 16: The measures of scale transformation and ranking the FLPs by our model.

FLAs	TC	RC	Socres	Rank
A1	1	0	0.7	3
A2	0.9332	0.1730	0.7051	2
A3	0.6050	1	0.7235	1
A4	0.2569	0.09943	0.2096	4
A5	0	0.0430	0.0129	5

criteria. We first use a mathematical programming model to generate several feasible alternatives, these alternatives have been ranked by the transportation cost. Then we develop a novel MAGDM method to get the rankings considering their qualitative attributes. The experts' evaluation scores on the alternatives are in the form of IFNs, and we use the cosine angle value to determine the consensus degree among experts. We assume each expert has different weights on attributes, which are partially known or completely unknown. An illustration example is also presented in the paper.

Future works can be conducted in several ways, followings are some of them:

- A tricky issue is the decision on the number m of the feasible layout alternatives and the methodology to choose them from the feasible solution set. Although we can sort them by transportation costs, but it turns out that similar layout solutions may be produced. In this paper we just select every other solution to reach the diversity, one specified method deserves to be developed.
- In order to minimize the inconformity of ranking habit of different experts, we combine the method of cosine angle value and Geometric mean (which performs better than Arithmetic mean) instead of Euclidean distance. A better methodology is being considered.
- The layout problem with irregular area facility or departments can be considered, with/without some forbidden areas or pre-determined departments.

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