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xVA: Definition, Evaluation and Risk Management

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Abstract The expected present values of the compensation for counterparty default risk and the funding costs under the risk-neutral measure are defined to be the bilateral CVA and FVA, respectively. The latter breaks down into FCA, MVA, ColVA and KVA. The market funding liquidity risk, can be bilaterally priced into a derivative trade, without causing price asymmetry between the counterparties.

Keywords Counterparty credit (or default) risk (CCR), Bilateral replication, Credit valuation adjustment (CVA), Funding valuation adjustment (FVA), Funding cost adjustment (FCA), Margin valuation adjustment (MVA), Collateral valuation adjustment (ColVA), Capital valuation adjustment (KVA)

1. Introduction

The 2008 financial crisis had catalyzed many changes to the global derivatives market which, in particular, are seen through the proliferation of collaterals, the mandate to trade all derivatives through central counterparty clearing houses (CCP), and the emergence of various valuation adjustments, so-called xVA, in either pricing or accounting. The xVA contains a growing list of valuation adjustments (VAs): credit valuation adjustment (CVA), debit valuation adjustment (DVA), funding valuation adjustment (FVA), funding cost adjustment (FCA), collateral valuation adjustment (ColVA), margin valuation adjustment (MVA), capital valuation adjustment (KVA), and etc. Yet, except for CVA and DVA[1], the other VAs have been controversial because, in addition to a lack of unanimous definition, their inclusion in pricing or booking causes price asymmetry and asset-liability asymmetry to the trading parties. Moreover, in a poll conducted by Risk.net in March 2015 (Sherif, 2015), about two thirds of quants believe that banks have overstated FVA losses and the current FVA model is wrong. In this article, we will redefine the notions of CVA, DVA and FVA and derive the formulae for the rest of xVA as the expected present values of excessive cash flows due to funding spreads under the risk-neutral pricing measure. We then identify the component of the xVA to be included into the fair price defined in IFRS 13 (i.e., exit price or market price, see FASB (2011)). In the end we will make our suggestions to managing the unhedgeable idiosyncratic risks behind the rest of xVA or xVA components.

To tackle funding costs in generality, a number of models have been developed, including Brigo *et al.* (2011), Crépay (2011), Burgard and Kjaer (2011), Lou (2015), Bichuch *et al.*

[1] DVA is the CVA of the counterparty.

(2015) and Li and Wu (2015). Burgard and Kjaer (2011), in particular, pioneer the arbitrage pricing of derivatives subject to market risks, counterparty default risks and funding costs/risks, and obtain the triad of valuation adjustments (i.e., CVA , DVA and FVA) under a consistent framework. While these models help us to gain insights on margin and collateral management, they contribute little on resolving the controversy surrounding the FVA: price asymmetry to the trading parties and asset-liability asymmetry in accounting, which are required by the rule of IFRS 13 (IASB, 2012).

In this article, we will price general derivatives and quantify the xVA in two steps. First, we identify the risk-neutral pricing measure and then uncover the corresponding hedging or replicating strategies against the market risks and the counterparty default risks. Note that these strategies are unaffected by the funding spreads and are to be taken bilaterally by the counterparties. The cost of the replications in the absence of funding spreads is identified to be nothing else but the risk-neutral value of the derivative. Second, we will figure out the additional costs caused by the funding spreads (for margins/collaterals, capitals as well as for hedging) and, following the market convention, define the FVA as the expected present values of the additional costs under the risk-neutral measure. Out of the FVA we further identify FCA, MVA, ColVA and KVA as components. A major finding of this paper is that the market funding risk premium for unsecured lending and borrowing can be priced in into trade prices yet will not cause price asymmetry to the counterparties, in line with the emerging market practice to charge a uniform market funding liquidity risk premium in derivatives trades.

2. Pricing through bilateral replications

2.1. Margin accounts, collaterals and capitals

Without loss of generality, we consider a partially collateralized derivative trade between two defaultable parties, B a bank and C a counterparty. If there is no default by either party until the maturity of the derivative, the party of liability will make a contractual payment to the counterparty. In case of a premature default, the party of exposure will seize the collateral, and will remain entitled for a fair share of recovery values. For subsequent discussions, we introduce the following notations.

Y_T — the contractual payoff of the derivative at maturity T,
τ_i — the default time of party i, for $i = B$ and C,
$I_i(t)$ — ≥ 0, the value of the initial margin (IM) posted by party i,
$X_i(t)$ — ≥ 0, the value of the variable margin (VM) or collaterals posted by party i,
$K_i(t)$ — ≥ 0, the value of risk capitals allocated to party i by shareholders,
$V(t)$ — the fair value of the derivative to B, the bank, s.t.
$V(t) > 0$ — asset, $V(t) < 0$ — liability.

The margins, collaterals and capitals are typically in the form of cash or other Tier 1 capital assets, which cost fees to borrow and thus incurs funding costs. Collaterals and variable margins may be lent out by the receiving party for its own funding purposes, which is called rehypothecation, and thus may be rewarded with higher returns. For details of margin and collateral management, we refer to, e.g., Gregory (2015) or Green (2016).

Across the moment of the first bilateral default, $\tau = \tau_B \wedge \tau_C$, there may be a downward jump in the derivatives value. For pricing purpose, we need to specify the post-default value of the derivative by using either the Standardized Method or the Internal Model Method (IMM), see also Gregory (2015) or Green (2016).

For both mathematical or notational simplicity, we make two non-essential assumptions: 1) the risk-free rate and default intensities are deterministic functions of time, and 2) the credit default swap (CDS) rates remain constants. Also, we assume there is no default at $t = 0$, the current moment.

2.2. Pricing in the absence of funding cost

We will model the market with probability space $(\Omega, \mathcal{G}, \mathcal{G}_t, \mathbb{P})$, where $\mathbb{P}$ is the real-world measure, $\mathcal{G}_t$ is the filtration that represents all market information up to time t, such that $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t$, where $\mathcal{F}_t$ contains all market information except the default statuses of the trading parties, while $\mathcal{H}_t$ contains only the information of default statuses of the counterparties.

Without loss of generality, we consider the pricing of equity European options in a market where the following securities are available to trades: the money market account, shares or stocks, and CDS. Let B_t, S_t and $U_i(t), i = B$ and C denote the corresponding values of prices of these securities, then their dynamics are described by

$$dB_t = r_t B_t dt, \quad \text{with} \quad B_0 = 1, \tag{1}$$

$$dS_t = S_t \left[\mu_t dt + \sigma_t dW_t^{(P)}\right], \tag{2}$$

$$dU_i(t) = r_t U_i(t) dt - s_i dt + L_i dJ_i^{(P)}, \quad i = B \text{ and } C, \tag{3}$$

where r_t is the risk-free short rate, μ_t is the expected return, and σ_t is the percentage volatility, $W_t^{(P)}$ is a one-dimensional Brownian motion under $\mathbb{P}$, $s_i \geq 0$ is the annualized CDS rate, $J_i^{(P)}$ is a Poisson process that jumps from 0 to 1 upon the default of party i with the $\mathbb{P}$ intensity of the jump is $\lambda_i^{(P)}$, and $L_i \in [0,1]$ is the corresponding loss rate, assumed to be a constant for simplicity. The notional value of the CDS is set to one dollar. For hedging against counterparty risk, we will always make use of the par CDS, which has zero value upon dynamical hedge revisions.

The shares for hedging purpose will be "repoed in", such that the shares will be used as collaterals for borrowing. The instantaneous return from holding the repo is

$$dZ_S(t) = dS_t - (r_t + \lambda_S - q_t) S_t dt = S_t \left[(\mu_t - r_t - \lambda_S + q_t) dt + \sigma_t dW_t^{(P)}\right],$$

where the second term on the RHS of the first equality is the cost of carry for the repo trade, with q_t being the dividend yield of the share and λ_S being the repo spread[2]. For a repo entered at time t, its value equals to zero: $Z_S(t) = 0$.

According to the fundamental theory of asset pricing (Harrison and Pliska, 1981), there exists a measure, $\mathbb{Q}$, which is equivalent to $\mathbb{P}$ such that under $\mathbb{Q}$ the discount prices of the repo and the CDS are martingales. For our asset price model, such a martingale measure is uniquely defined by the following Radon-Nikodym derivative:

$$\begin{aligned} \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} &= \frac{e^{-\int_0^t \gamma_S(u) dW_u^{(P)}}}{E_0^P\left[e^{-\int_0^t \gamma_S(u) dW_u^{(P)}}\right]} \frac{e^{\gamma_B J_B^{(P)}(t)}}{E_0^P\left[e^{\gamma_B J_B^{(P)}(t)}\right]} \frac{e^{\gamma_C J_C^{(P)}(t)}}{E_0^P\left[e^{\gamma_C J_C^{(P)}(t)}\right]} \\ &= e^{\int_0^t -\frac{1}{2}\gamma_S^2(u) du + \gamma_S(u) dW_u^{(P)} + \gamma_B J_B^{(P)}(t) + \lambda_B^{(P)} t(1-e^{\gamma_B}) + \gamma_C J_C^{(P)}(t) + \lambda_C^{(P)} t(1-e^{\gamma_C})}, \end{aligned} \tag{4}$$

with

$$\gamma_S(t) = \frac{\mu_t - r_t - \lambda_S + q_t}{\sigma_S}, \quad \gamma_B = \ln \frac{s_B / L_B}{\lambda_B^{(P)}} \quad \text{and} \quad \gamma_C = \ln \frac{s_C / L_C}{\lambda_C^{(P)}}.$$

Under $\mathbb{Q}$ the price dynamics of the repo and the CDS become

$$\begin{aligned} dZ_S(t) &= \sigma_S(t) S_t dW_t^{(Q)}, \\ dU_i(t) &= r_t U_i(t) dt + L_i \left(dJ_i^{(Q)} - \lambda_i^{(Q)} dt\right), \quad i = B \text{ and } C, \end{aligned} \tag{5}$$

[2] For simplicity, we have skipped discussing the details of haircuts in repo trades for long or short stocks, which can be easily accommodated by adjusting the repo spread.

where $W_t^{(Q)}$ is a $\mathbb{Q}$-Brownian motion and $J_i^{(Q)}$ is a jump process with risk-neutral intensity $\lambda_i^{(Q)} = s_i/L_i$. Apparently, both $Z_S(t)$ and the discount prices of $U_i(t)$ are $\mathbb{Q}$-martingales. We call $\mathbb{Q}$ the risk-neutral measure, and denote $E_t^Q[X]$ for $E^Q[X|\mathcal{G}_t]$ for notational simplicity. We have the following result on the arbitrage-free valuation of the derivative.

Proposition 2.1. The risk-neutral valuation of the derivative to the counterparties is

$$V_f(0) = E_0^Q\left[e^{-\int_0^{\tau\wedge T} r_s ds} V(\tau\wedge T)\right] \quad \square \tag{6}$$

When Y_T is $\mathcal{F}_T$-adaptive instead of $\mathcal{G}_T$-adaptive, the risk-neutral valuation (**??**) contains the bilateral credit valuation adjustments (BCVA) as price components. In fact, there is

Corollary 2.1. When Y_T is $\mathcal{F}_T$-adaptive, the risk-neutral valuation of the derivative has the following decomposition:

$$V_f(0) = V_e(0) + \mathrm{CVA}_B + \mathrm{CVA}_C, \tag{7}$$

where

$$\begin{aligned} V_e(0) &= E_0^Q[e^{-\int_0^T r_s ds} Y_T], \\ \mathrm{CVA}_B &= E_0^Q[1_{\{\tau=\tau_B\le T\}} e^{-\int_0^{\tau_B} r_s ds}(V(\tau_B) - V_e(\tau_B))], \\ \mathrm{CVA}_C &= E_0^Q[1_{\{\tau=\tau_C\le T\}} e^{-\int_0^{\tau_C} r_s ds}(V(\tau_C) - V_e(\tau_C))] \quad \square \end{aligned} \tag{8}$$

Being $\mathcal{F}_T$-adaptive means Y_T is always definable, regardless the default statuses of the counterparties. An option on a stock index is one example.

We gain two insights from formulae (**??**) and (**??**). First, the bilateral CVA is part of the risk-neutral valuation, which is symmetrical to the counterparties[3]. Second, the no-default value (NDV) of the derivative is the only right choice for the MtM value of the derivative when it comes to calculating the LGD.

We have the following interpretations for NDV and CVAs.

Proposition 2.2. In the absence of funding cost, $V_e(0)$ is the present value (PV) of cost to replicate the payoff, and CVA_i is the $\mathbb{Q}$-expected present value of the cost to replicate the LGD by party i.

In the absence of funding spreads, we can also show that there is always

$$E_0^Q\left[e^{-\int_0^{\tau\wedge T} r_s ds}\beta_i(\tau\wedge T) - \beta_i(0)\right] = 0, \quad \text{for } i = B \text{ and } C, \tag{9}$$

regardless of what price is taken for the trade, which is not the case in the presence of funding spreads.

3. The rise of other xVA

We now take the funding spreads for margins, collaterals and capitals into account. Once B and C enter into a trade at time $t = 0$ for a value V_0 to B, the two parties start hedging, using repos and/or CDS, posting margins/collaterals and setting aside capital according to margin, collateral and capital rules. The hedging portfolios of the two parties can be expressed as

$$\begin{aligned} \Pi_B(t) &= \delta_B Z_S(t) + \alpha_B U_C(t) + \beta_B(t), \\ \Pi_C(t) &= \delta_C Z_S(t) + \alpha_C U_B(t) + \beta_C(t), \end{aligned} \tag{10}$$

[3] From the perspective of B, CVA_C is the CVA, while CVA_B is the DVA. The combined value of CVA_B and CVA_C is the bilateral CVA.

where δ_i is the number of repo contracts held by party i, which can be

$$\delta_B = -\frac{\partial V_e(t)}{\partial S}, \quad \delta_C = \frac{\partial V_e(t)}{\partial S},$$

and α_i is the numbers of CDS contract taken up by party i to hedge against LGD:

$$\alpha_B(t) = -\frac{V(\tau_C = t) - V_e(t)}{L_C}, \quad \alpha_C(t) = -\frac{V(\tau_B = t) - V_e(t)}{L_B},$$

and $\beta_i(t)$ is the total value of party i's cash in his saving account, with initial values

$$\beta_B(0) = \Pi_B(0) = -V_0, \quad \text{and} \quad \beta_C(0) = \Pi_C(0) = V_0.$$

Here, V_0 is the initial premium payment paid by or received by B, depending on whether it is positive or negative.

The saving accounts serves a number of purposes:

(1) Saving or unsecured borrowing.
(2) To pay for the funding costs for IM, VM and capital.
(3) To take the P&L for hedging.

In addition, at any moment, a portion, $\phi_B(t) \in [0, 1]$, of capital borrowed from share holders may be reallocated to the saving account. To describe the evolution of the balance of the saving account, we need the following notations:

x_B	—	the spread for unsecured borrowing or lending for party i,
$x_B^{(I)}$	—	the funding spreads over the risk-free rate for initial margin,
$x_B^{(X)}$	—	the funding spreads over the risk-free rate for VM or collateral,
$\gamma_B^{(K)}$	—	the funding spreads over the risk-free rate for capital (so-called net return of the capital to the share holders),
$\phi_B(t)K_B(t)$	—	the portion of capital allocated to the saving account,
$\delta_B dZ_S(t)$	—	the P&L for delta hedging using repos over $(t, t+dt)$, and
$\alpha_B dU_C(t)$	—	the P&L for hedging CCR with the CDS over $(t, t+dt)$.

Then, the change of the saving account over the time interval $(t, t+dt)$ can be described as

$$\begin{aligned} d\beta_B(t) = \Big(&(r_t + x_B)[\beta_B(t) + \phi_B(t)K_B(t)] - x_B^{(I)} I_B(t) - x_B^{(X)} X_B(t) \\ &-[\gamma_B^{(K)} + \phi_B(t) r_t] K_B(t) \Big) dt + \delta_B dZ_S(t) + \alpha_B dU_C(t), \end{aligned} \tag{11}$$

for which we need to make additional elaborations.

(1) The rate of return for the saving account can be nonlinear and asymmetric, such that

$$x_i(\beta_{m,i}(t)) = x_m + x_i^{(l)} 1_{\{\beta_{m,i}(t) \geq 0\}} + x_i^{(b)} 1_{\{\beta_{m,i}(t) < 0\}}.$$

Here, $x_m \geq 0$ is the market funding liquidity risk premium, $x_i^{(b)} \geq 0$ is the default risk premium for party i, and $x_i^{(l)} \geq 0$ is the default risk premium of the borrower. In general, there is $x_i^{(b)} \neq x_i^{(l)}$.

(2) Due to a partial allocation of capital to the saving account, the interest accrual of the saving account becomes $(r_t + x_B)[\beta_B(t) + \phi_B(t)K_B(t)]$, in an expense of $r_t \phi_B(t) K_B(t) dt$.

(3) In general, the funding spreads and the return on capital are positive, reflecting a simple reality that costs will be incurred for borrowing funds or capitals. Yet for VM or collaterals, there can be exceptions in case of rehypothecation. When the return from rehypothecation is higher than the borrowing cost, the corresponding spread turns negative, representing a funding benefit to the party who posts VM or collaterals[4].

[4] It should be pointed out, however, the higher return is often due to the exposure to credit risk. Thus, for pricing purpose, one may have to adjust the spread properly to account for the credit risk.

(4) When the derivative is an asset, there will be a jump in the balance of the saving account upon counterparty default due to the the CDS payment for LGD.

We now analyze the P&L of the trade to party B. The present value of the P&L is simply the PV of B's hedged portfolio's value at the termination time of the option (upon either default or maturity):

$$P\&L = e^{-\int_0^{\tau\wedge T} r_s ds}\left[\beta_B(\tau\wedge T) + V(\tau\wedge T)\right].$$

Due to the perfect replication of the market risk and the elimination of LGD by the CDS, there is the equality

$$\begin{aligned} V(\tau\wedge T) &= V_e(\tau\wedge T) - \alpha_B L_C dJ_C^{(Q)}(t) \\ &= V_e(0)e^{\int_0^{t\wedge T} r_v dv} - \int_0^{t\wedge T} e^{\int_u^{t\wedge T} r_v dv}\delta_B(u)dZ_S(u) - \alpha_B L_C dJ_C^{(Q)}(t). \end{aligned}$$

We then rewrite the P&L into

$$\begin{aligned} P\&L = &\left[(\beta_B(0) + V_e(0)) - \int_0^{t\wedge T} e^{-\int_0^u r_s ds}\alpha_B(u)s_C du\right] \\ &+ \left[e^{-\int_0^{t\wedge T} r_s ds}\beta_B(\tau\wedge T) - \beta_B(0) - \int_0^{t\wedge T} e^{-\int_0^u r_s ds}\delta_B(u)dZ_S(u)\right. \\ &\left. - \int_0^{t\wedge T} e^{-\int_0^u r_s ds}\alpha_B(u)dU_C(u)\right] \triangleq I + II. \end{aligned}$$

where I and II represent the terms enclosed by the two pairs of square brackets, respectively. We will show next that I represents the PV of the shortfall of payout replication, and II represents the PV of accumulative costs due to various funding spreads. Using the result of Proposition 2.2, we have

$$E_0^Q[I] = \beta_B(0) + V_f(0) - CVA_B.$$

If we take the trade price to be $V_0 = V_f(0) = -\beta_B(0)$, then there is

$$E_0^Q[I] = -CVA_B,$$

meaning that the debit valuation adjustment for party B is part of his P&L, and the DVA to B will result in a replication shortfall. For II we have

Proposition 3.1. The realized funding cost to B is

$$\begin{aligned} II = \int_0^{\tau\wedge T} e^{-\int_0^t r_s ds}\Big(&x_B[\beta_B(t) + \phi_B(t)K_B(t)] - x_B^{(I)}I_B(t) \\ &- x_B^{(X)}X_B(t) - \gamma_B^{(K)}K_B(t)\Big)\,dt. \end{aligned} \tag{12}$$

Note that II is a path-dependent random variable, except for the case of zero funding spreads for margins, collaterals and capitals, when there is $II \equiv 0$, regardless the price V_0 taken for the trade. Following the market convention, we define FVA as the expectation of the present value of realized funding cost under the risk-neutral measure.

Definition 3.1. The FVA to party B is the expected value of excess cost due to the funding spreads for various funding transactions under the risk-neutral measure:

$$\mathrm{FVA}_B = E_0^Q\left[II\right].$$

From the definition of FVA we immediately have

Proposition 3.2. To party B, the funding valuation adjustment is given by

$$\mathrm{FVA}_B = \mathrm{FCA}_B + \mathrm{MVA}_B + \mathrm{ColVA}_B + \mathrm{KVA}_B, \tag{13}$$

with

$$\begin{aligned}
\mathrm{FCA}_B =& E_0^Q\left[\int_0^{T\wedge\tau} x_B e^{-\int_0^t r_s ds}[\beta_B(t)+\phi_B(t)K_B(t)]dt\right],\\
\mathrm{MVA}_B =& -E_0^Q\left[\int_0^{T\wedge\tau} x_B^{(I)} e^{-\int_0^t r_s ds} I_B(t)dt\right],\\
\mathrm{ColVA}_B =& -E_0^Q\left[\int_0^{T\wedge\tau} x_B^{(X)} e^{-\int_0^t r_s ds} X_B(t)dt\right],\\
\mathrm{KVA}_B =& -E_0^Q\left[\int_0^{T\wedge\tau} \gamma_B^{(K)} e^{-\int_0^t r_s ds} K_B(t)dt\right],
\end{aligned} \tag{14}$$

and $\beta_B(t)$ evolving according to (**??**) □

We want to emphasize here that in general FVA and FCA can depend on V_0, the trade price of the derivative, through $\beta_B(0) = -V_0$. Hence, whenever necessary, we will write $\mathrm{FCA}(-V_0)$ or $\mathrm{FVA}(-V_0)$ to highlight such dependence.

Different choices of V_0 will lead to some major existing results on xVA. A popular choice of V_0 is the "value to me" (Ruiz, 2013), which is defined to be the value with which the risk-neutral expected return of the trade is the risk-free rate. The "values of the derivative to the parties", $V_0^{(B)}$ and $V_0^{(C)}$, satisfy (Li and Wu, 2015)

$$V_0^{(B)} = V_f(0) + \mathrm{FVA}(-V_0^{(B)}) \quad \text{and} \quad -V_0^{(C)} = -V_f(0) + \mathrm{FVA}_C(V_0^{(C)}).$$

which are implicit functions for $V_0^{(B)}$ and $V_0^{(C)}$, the so-called bid and ask prices in Li and Wu (2015). Normally, there is a price gap between the bid and ask prices, due to the idiosyncratic nature of funding costs.

We now highlight that we can achieve asset-liability asymmetry when idiosyncratic funding risk premiums are ignored. To see this, we put all funding spreads but x_m to zero, then the interest rates for unsecured lending and borrowing both become $\hat{r}_t = r_t + x_m$, which is the de facto risk-free rate. By repeating the arguments in section 2.2 for the risk-neutral valuation and replication pricing, we can obtain the fair value to the counterparties, as detailed in the following proposition.

Proposition 3.3. Assume independence between market funding risk and counterparty default risks. When all but the systematic (or market) funding risk premium are ignored, the bid and ask prices of a derivative are identical and are given by

$$\hat{V}_f(0) = \hat{V}_e(0) + \widehat{CVA}_B + \widehat{CVA}_C, \tag{15}$$

where

$$\begin{aligned}
\widehat{V}_e(0) &= E_0^{\hat{Q}}[e^{-\int_0^T \hat{r}_s ds} Y_T],\\
\widehat{CVA}_B &= E_0^{\hat{Q}}[1_{\{\tau=\tau_B\le T\}} e^{-\int_0^{\tau_B} \hat{r}_s ds}(V(\tau_B) - \widehat{V}_e(\tau_B))],\\
\widehat{CVA}_C &= E_0^{\hat{Q}}[1_{\{\tau=\tau_C\le T\}} e^{-\int_0^{\tau_C} \hat{r}_s ds}(V(\tau_C) - \widehat{V}_e(\tau_C))],
\end{aligned} \tag{16}$$

and $\hat{\mathbb{Q}}$ is the martingale measure corresponding to the numeraire asset $\hat{B}_t = e^{-\int_0^t \hat{r}_s ds}$ □

The implication of the last Proposition is that $\widehat{V}_f(0)$ is the (only) candidate for the fair price of the derivative to the counterparties. Note that in practice, we have seen that banks are converging to using a uniform "market cost of funding" to price derivatives (Gregory, 2015).

4. Conclusions

In this article, we have extended the notion of replication pricing and identified the components of xVA under the risk-neutral valuation. We have reached the conclusions that the fair price of a derivative is the classical risk-neutral value in a market where the rate for unsecured lending and borrowing is the risk-free rate plus the market funding liquidity risk premium, and such a value naturally contains the bilateral CVA. The fair price so defined is symmetric to the counterparties, and should be taken for both trading and booking. The idiosyncratic funding risks still have impacts on the P&L of derivative trades, and thus need to be managed. We have shown in this paper that the funding cost (or its components) can be expressed as path integrals in terms of hedge ratios, margin/collateral levels and capital level, and thus can be simulated. As such, we can apply a risk measure like VaR or CVaR under the physical measure to gauge the funding risks in a conventional way. Based on the value of the VaR or CVaR, we can determine the proper amount of IM, VM or capital for risk management purposes.

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