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Risks of Supply Disruption: An Evaluation of an Options Contract for Backup Supply

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Abstract In this paper, we consider the design of an options contract for a manufacturer using backup supply who is facing risks of supply disruption. We derived optimal parameters of the options contract and carried a numerical analysis which mainly focussed on the manufacturer' choice between 'responsive' and 'low-cost' supplier, and supplier opportunism and incentives. We observed that uncertainty of any kind forces the manufacturer to utilize more of its backup options, however, the extent of such influence varies based on the nature of the uncertainty. The backup supplier exploits the situation by increasing its prices.

Keywords options contract, backup supply, supply chain, stackelberg game

1. Introduction

When firms sources from low-cost countries like, China, dependence only on the offshore supplier for all supply requirements exposes the supply chain to a higher level of risks. Such dependence provides no flexibility and may cause the whole supply chain dysfunctional in the event of any disruption to the supply network. For example, Ericsson incurred a revenue loss of approximately $400 million due to a fire that forced Phillips, its only source, to shut down its semiconductor plant at Albuquerque, New Mexico for six weeks (Latour, 2001). Similarly, Toyota had to shut down 20 assembly lines (out of 40) for 6 weeks because of a fire at one of its suppliers supplying brake-fluid adjusting valves. The loss incurred by this shut down was estimated at $40 million each day (Nishiguchi and Beaudet, 1998). Thus, the above events cannot be overlooked as the consequences involved are very high. A practical approach to mitigate such risks of disruption would be to use a backup supplier who is responsive enough to deliver shortages due to such disruption within a very short period. Often a variety of supply contracts like, options contract, capacity reservation contracts, utilization-dependent price-only contract etc. are being used for incentivizing backup supplier while retaining the flexible option. We, in this paper, evaluate the use of an options contract for a manufacturing firm using a backup supply option.

2. LITERATURE REVIEW

A large number of researchers has focused on evaluating an inventory system when it faces the risk of disruption of the supply. Atasoy et al. (2012) has considered all-or-nothing type of

supply situation and have shown the optimality of the (s, S) inventory policy when customer demand is deterministic and non-stationary over time. Chen and Xiao (2015) considered a production disruption and outsourcing scenario and analysed the manufacturer-retailer game to draw insights to the retailer's outsourcing decision. Although there are literature on inventory systems with supply disruption for both continuous (Snyder, 2014; Parlar, 1997) and periodic review systems (Song and Zipkin, 1996; Li et al., 2004; Parlar et al., 1995; Gallego and Hu, 2004; Chen et al., 2012), however, a news-vendor type analysis dominates. As a mitigation strategy researchers have favored using multiple sources of supply. Minner (2003) gives a very good review of the literature on inventory models involving multiple sources of supply. Zhu (2015) evaluated the benefit of advance information about supply disruption in a multiple-sourcing context.

There is a significant merit in developing long-term supplier relationships with few key suppliers but such a strategy further exposes itself to risks of disruption. The use of a backup supplier is a mid-path between leveraging long-term partnerships and hedging against disruption risks. The use of supply contracts to achieve flexibility in supply chain galore. Kouvelis and Li (2008) have shown that the use of a backup supplier with a shorter lead time and an emergency response option can improve performance of a inventory replenishment system. Hou et al. (2010) evaluated a buy-back contract when the supplier faces the risk of supply disruption. Zeng and Xia (2015) analysed on the benefits of using a backup supplier under supply disruption and determined optimal contract parameters for the situation. Sting and Huchzermeiyer (2010) has shown that, under forced compliance regime, a supply options coordinate the supply chain involving a backup supplier and a main supplier subjected to supply disruption risks whereas firm commitment contract leads to losses. They further showed that, under voluntary compliance regime, a price-only contract fails to achieve coordination but a utilization-dependent price-only contract. We, in this paper, consider the use of options contract for a backup supplier used as a mitigation strategy against supply disruption.

3. THE MODEL

We consider a situation where a manufacturer engages a low-cost supplier for sourcing a major part or components and one backup supplier for meeting its short-term supply shortage. The manufacturer places an order of quantity Q_M (at a unit price of w_M) with the main supplier which is supposed to meet supply requirements for the planning horizon of length T. The manufacturer expects to receive the material from the main supplier after L periods and also faces the uncertainty of a sudden disruption of its supply with a probability of occurrence of such disruptions as θ. At $t = 0$, the manufacturer also signs an options contact with the backup supplier for a quantity of Q at price w_o. The backup option is exercised by the manufacturer only once and at $t = L$ and pays an exercise price of w_e when it exercises the options. The manufacturer's decision problem can be divided into two parts: (1) at $t = 0$, the manufacturer decides its order quantity with the main supplier (i.e. Q_M), and the quantity of options to purchase from the backup supplier (i.e. Q), and (2) at $t = L$, however, the manufacturer decides as to how much of the options to exercise (denoted as q). We refer to the former as Period 1 problem while the later as Period 2 problem. We consider a forced compliance regime for the supplier and hence, on signing the options contract for Q units the supplier builds an inventory of K units such that $K \leq Q$. The supplier acquires the K units at $t = 0$ is c per unit. At $t = L$, however, the supplier may have to exercise its expedite option when the manufacturer's order quantity q exceeds K. The supplier's expedite cost is c_1 such that $c_1 > c$. The supplier incurs an holding cost of h_s per unit for any leftover inventory. We refer to the expected profit function of the the supplier as $J(w_o, w_e)$ and can be expressed as,

$$J(w_o, w_e) = w_o Q + w_e E\{min(Q, q)\} - h_s E\{[K - q]^+\} - c_1 E\{\min([q - K]^+, Q - K)\} - cK \quad (1)$$

Hence, the supplier has two decisions to make: (1) prices w_o and w_e, and (2) the initial inventory to build i.e. K. The supplier will take its pricing decisions based on its estimation of what might be the manufacturer's optimal quantity decisions.

4. MANUFACTURER'S OPTIMAL DECISIONS

We present the analysis of the manufacturer's Period 2 first followed by the analysis of Period 1 problem.

4.1. PERIOD 2 ANALYSIS

Let at $t=0$, the manufacturer begins with an on-hand inventory of x_0 and targets to build an inventory of y through exercising its options with the backup supplier. Let, $x_1=[x_o-d_1]^+$, and $x=x_1+Q_M$, if the main supplier does not fail. If the main supplier fails then $x_1=[x_0-d_1]^+$, and $x=x_1$. The manufacturer decides the exercise quantity so as to minimize the sum of options exercise price, expected penalty cost (p) due to shortages at $t=T$, and expected inventory holding cost (h) due to overstock at $t=T$. Hence, the manufacturer's cost function at this stage can be stated as,

$$C_2(y)=w_e(y-x)+p\int_y^{\infty}(\xi_2-y)f_2(\xi_2)d\xi_2+h\int_0^{y}(y-\xi_2)f_2(\xi_2)d\xi_2 \tag{2}$$

As (2) is convex, from the first order condition, we can determine the optimal policy as for the manufacturer as stated in Lemma 1.

Lemma 1.
The optimal policy of the manufacturer's Period 2 decision problem is

$$q^*=\begin{cases} Q; & 0\le x<S_2-Q \\ S_2-x; & S_2>x\ge S_2-Q \\ 0; & x\ge S_2 \end{cases}$$

The parameter, S_2 is determined as, $S_2=F_2^{-1}\left(\frac{p-w_e}{p+h}\right)$.

4.2. PERIOD 1 ANALYSIS

In this section, we model the manufacturer's decision problem at $t=0$. As mentioned earlier, at $t=0$, the manufacturer faces the following decision: (1) decide the order quantity to be placed with the backup supplier, Q_M, and (2) decide the optimal amount of options to buy from the backup supplier, Q. The manufacturer takes its decision in a way that minimizes its total cost $C(Q,Q_M)$ which can be expressed as below.

$$C(Q,Q_M)=w_oQ+\theta\int_0^{\infty}C_2([x_0-\xi_1]^+,Q)f_1(\xi_1)d\xi_1+(1-\theta)\int_0^{\infty}C_2([x_0-\xi_1]^++Q_M,Q)f_1(\xi_1)d\xi_1 \tag{3}$$

Define,

$$r=\begin{cases} Q; & \text{if} \quad Q+Q_M<S_2 \\ S_2-[x_0-d_1]^+-Q_M; & \text{if} \quad Q+Q_M\ge S_2 \end{cases}$$

$$R=\begin{cases} [x_0-d_1]^++Q+Q_M; & \text{if} \quad Q+Q_M<S_2 \\ S_2; & \text{if} \quad Q+Q_M\ge S_2 \end{cases}$$

and

$$m=\begin{cases} 1; \text{ if} & Q+Q_M<S_2 \\ 0; \text{ if} & Q+Q_M\ge S_2 \end{cases}$$

$$m' = \begin{cases} 0; & \text{if} \quad Q + Q_M < S_2 \\ -1; & \text{if} \quad Q + Q_M \geq S_2 \end{cases}$$

We can now determine the optimal decision for the manufacturer in the form of first order condition and is stated in Lemma 2.

Lemma 2.
(a) The functions $C(Q, Q_M)$ is jointly convex in (Q, Q_M).

(b) The minimizers of $C(Q, Q_M)$, Q^* and Q_M^* satisfies the following relations

$$w_o + \theta \int_{[x_0 - S_2 + Q]^+}^{\infty} \Big\{(w_e - p) + (p + h)F_2([x_0 - d_1]^+ + Q)\Big\} f_1(d_1) dd_1 + (1 - \theta) \int_{[x_0 - [S_2 - Q - Q_M]^+]^+}^{\infty} m\Big\{(w_e - p) + (p + h)F_2(R)\Big\} f_1(d_1) dd_1 = 0 \tag{4}$$

and

$$w_M - w_e + \int_{0}^{[x_0 - S_2 + Q_M]^+} \{(w_e - p) + (p + h)F_2(x_0 - d_1 + Q_M)\} f_1(d_1) dd_1 + \int_{[x_0 - [S_2 - Q - Q_M]^+]^+}^{\infty} m\{(w_e - p) + (p + h)F_2(R)\} f_1(d_1) dd_1 = 0 \tag{5}$$

: Claim (a) can be proved by showing that the Hessian is positive semi-definite and claim (b) can be obtained from the first order conditions. Q.E.D.

5. THE MANUFACTURER-SUPPLIER GAME

We now formulate the backup supplier's decision problem as a Stackelberg game where the supplier plays the role of a leader and the manufacturer plays the role of a follower. The supplier takes its decision in a way that maximizes its profit $J(w_o, w_e)$ as given in (1). However, the supplier's pricing decision influences the manufacturer's options buy decision i.e. Q. As the supplier decides to charge a higher price (which eventually may increase the supplier's profit) the manufacturer reacts by lowering its options buy quantity and increasing its order with the main supplier. Thus, a game is played between the manufacturer and the supplier and therefore, our goal will be to determine an equilibrium solution for the Stackelberg game played between the two. Please note that the supplier takes its decision only at $t = 0$ and any shortfall in its inventory, in terms of fulfilment of the manufacturer's exercise request, is met through its responsive capacity available at a per unit cost of c_1. The optimal value for supplier's initial inventory K can be determined by using Lemma 3 as stated below.

Lemma 3.
The maximizer of $J(w_o, w_e)$, K^* satisfies the following condition

$$-c + \theta \left[-h_s \int_{0}^{[x_0 - S_2 + K]^+} f_1(d_1) dd_1 + c_1 \int_{[x_0 - S_2 + K]^+}^{\infty} f_1(d_1) dd_1 \right] + (1 - \theta) \left[-h_s \int_{0}^{[x_0 - [S_2 - Q_M - K]^+]^+} f_1(d_1) dd_1 + c_1 \int_{[x_0 - [S_2 - Q_M - K]^+]^+}^{\infty} f_1(d_1) dd_1 \right] = 0 \tag{6}$$

Now, supplier's optimal decision can be obtained by maximizing $J(w_o, w_e)$. It can be easily shown that : (1) the strategy space of the stackelberg game is compact, and (2) the manufacturer's cost function and the supplier's profit functions are supermodular. Hence, the equilibrium of the game exists. As the manufacturer's and the supplier's decision choices are bounded in $[0,U] \times [0,V]$, to determine the equilibrium point we first have to solve the constraint optimization problem and the KKT conditions are given in Lemma 4 and Lemma 5.

Lemma 4.
(a) If $Q^* = 0$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q} \mid_{Q=0} \geq 0$
(b) If $Q^* = U$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q} \mid_{Q=U} \leq 0$
(c) If $Q^* \in (0,U)$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q} = 0$
(d) If $Q_M^* = 0$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q_M} \mid_{Q_M=0} \geq 0$
(e) If $Q_M^* = U$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q_M} \mid_{Q_M=U} \leq 0$
(f) If $Q_M^* \in (0,U)$ *then* $\frac{\partial C(Q,Q_M)}{\partial Q_M} = 0$

Lemma 5.
(a) If $w_o^* = 0$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_o} \mid_{w_o=0} \leq 0$
(b) If $w_o^* = V$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_o} \mid_{w_o=V} \geq 0$
(c) If $w_o^* \in (0,V)$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_o} = 0$
(d) If $w_e^* = 0$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_e} \mid_{w_e=0} \leq 0$
(e) If $w_e^* = p$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_e} \mid_{w_e=p} \geq 0$
(f) If $w_e^* \in (0,p)$ *then* $\frac{\partial J(w_o,w_e)}{\partial w_e} = 0$

6. NUMERICAL EXPERIMENTS

In order to understand how the manufacturer and the backup supplier behave in a changing business environment, we conducted numerical experiments and observed how the equilibrium point of the manufacturer-supplier Stackellberg game changes with respect to the changes in the various parameters of the game. We take the base values for the parameters as given in Table 1.

TABLE 1. Parameter Values

Parameter	Base Value
Main supplier failure probability, *theta*	0.1
Mean demand for period 1, μ_1	10
Standard deviation of period 1 demand, σ_1	3.0
Mean demand for period 2, μ_2	20
Standard deviation of period 2 demand, σ_2	7.0
Parameters for the Manufacturer	
Shortage Cost, p	40mu (Monetary Unit)
Inventory Holding Cost at the end of period 2, h	3mu
Unit price for the supplies from the main supplier, w_M	21mu
Initial inventory, x_o	8
Parameters for the Supplier	
Unit of manufacturing for the supplier, c	6mu
Unit price of expedited supplies for the supplier, c_1	7mu
Inventory Holding Cost at the end of period 2, h_s	3mu

Uncertainty and Game Behavior

In the first set of experiments we want to understand how the backup supplier and the manufacturer will behave to the changes in the levels of uncertainty. The uncertainties that we considered are: (1) the failure probability of the supplies from the main supplier (i.e. θ), (2) demand uncertainty for period 1 demand (i.e. d_1), and (3) demand uncertainty for period 2 demand (i.e. d_2). Table 2 presents the effects of changing the failure probability of the main supplier on the equilibrium point. As observed in Table 2, when the probability of failure for the main supplier increases, the manufacturer uses more of the reliable backup supply option and the supplier exploits it by increasing its price. The manufacturers base stock quantity, on the other hand, increases due to reduction in options exercise price. However, such increase in the base stock led to increasing the order with the main supplier when the failure probability increases. At high failure probability, however, the manufacturer utilizes the backup supplier at higher level than the main supplier.

TABLE 2. Effect of change in the failure probability of the main supplier on the equilibrium point

Failure Probability (θ)	Options Quantity Q	Order with Main Supplier Q_M	Base Stock S_2	Options Buy Price w_o	Options Exercise Price w_e	Supplier's Stocking Quantity K
0.00	5.828	12.713	24.90	13.575	7.427	5.828
0.05	6.488	12.053	24.16	12.970	8.912	6.488
0.10	7.149	11.402	23.32	12.038	10.694	7.149
0.15	7.828	10.733	22.76	11.575	11.958	7.828
0.20	8.530	10.048	22.04	10.675	13.627	8.530
0.25	9.200	9.411	21.36	9.745	15.238	9.200
0.30	9.842	8.809	20.84	9.106	16.512	9.842

For analyzing the effects of changes in the demand uncertainty we changed both the standard deviation and the mean demand. Table 3-6 shows the effect of changes as explained in (a) and (b) above. With increase in σ_1 the distribution of the balance inventory level, x_1, at the end of period $t = L$ spreads out and hence, increases the uncertainty on the extent to which the options will be exercised. Note that the mean demand μ_1 is greater than the base value for the initial inventory level for the manufacturer i.e. $\mu_1 = 10 > x_0 = 8$. This leads to the observation that as σ_1 increases the uncertainty of the quantity $[x_0 - d_1]^+$. This in turn brings in more uncertainty with the inventory requirement at time $t = L$. Under such case, the manufacturer would like to use more of the backup option. The supplier exploits the situation and increases its offered options buy price. However, when σ_1 decreases there are more possibilities of d_1 being greater than x_0 and hence, the manufacturer does not benefit much by buying more options. This motivates the supplier to incentivize the manufacturer by offering a lower option buy price.

Contrast the above with the effects of changes in μ_1 as given in Table 4. Increase in μ_1 shifts the demand distribution further right of the initial inventory level, x_0 which increases the certainty of having a zero-inventory balance at time $t = L$. Hence, the manufacturer tends to shift a part of its orders from flexible to low cost options leaving no scope for playing opportunism by the supplier for increasing the price. In the opposite case, i.e. when μ_1 reduces and as it becomes more closer to x_0, uncertainty of x_1 increases. This motivates the manufacturer to use more of backup options. However, beyond a point the certainty of surplus inventory at the end of period 1 increases. Additional options quantity would, in such a case, would mostly be used for the mitigation of the uncertainty of the main supplier. From the manufacturer's perspective a lower options buy price would motivate more in favor of buying options as there are more uncertainty with x_1. The supplier responds by lowering the options buy price while increasing the exercise price.

Increase in σ_2 influences the manufacturers service level and hence, changes in σ_2 affects S_2. But for the dataset that we have chosen the effect of changes in σ_2 on S_2 is marginal

TABLE 3. Effect of changing σ_1 (the co-efficient of variation changes)

SD of Period 1 Demand (σ_1)	Options Quantity Q	Order with Main Supplier Q_M	Base Stock S_2	Options Buy Price w_o	Options Exercise Price w_e	Supplier's Stocking Quantity K
3.5	7.214	11.200	23.49	12.392	10.324	7.214
3.4	7.204	11.228	23.46	12.328	10.398	7.204
3.3	7.195	11.266	23.43	12.258	10.468	7.195
3.2	7.182	11.309	23.40	12.198	10.528	7.182
3.1	7.166	11.355	23.36	12.118	10.608	7.166
3.0	7.149	11.402	23.32	12.038	10.694	7.149
2.9	7.140	11.441	23.20	11.785	10.959	7.140
2.8	7.130	11.478	23.19	11.772	10.979	7.130
2.7	7.120	11.513	23.19	11.759	10.990	7.120
2.6	7.110	11.557	23.19	11.748	10.992	7.110
2.5	7.100	11.600	23.19	11.744	10.996	7.100

TABLE 4. Effect of changing μ_1 (the co-efficient of variation changes)

Mean Period 1 Demand (μ_1)	Options Quantity Q	Order with Main Supplier Q_M	Base Stock S_2	Options Buy Price w_o	Options Exercise Price w_e	Supplier's Stocking Quantity K
12.5	7.081	11.820	23.16	11.700	11.062	7.081
12.0	7.091	11.770	23.23	11.851	10.908	7.091
11.5	7.101	11.713	23.25	11.888	10.861	7.101
11.0	7.110	11.634	23.30	11.998	10.751	7.110
10.5	7.130	11.530	23.31	12.005	10.727	7.130
10.0	7.149	11.402	23.32	12.038	10.694	7.149
9.5	7.186	11.225	23.33	12.044	10.688	7.186
9.0	7.215	10.036	23.34	12.048	10.664	7.215
8.5	7.261	10.800	23.30	11.959	10.734	7.261
8.0	7.281	10.564	23.30	11.929	10.733	7.281
7.5	7.325	10.267	23.22	11.742	10.909	7.325

as shown in Table 5. However, the increase in uncertainty motivates the manufacturer to utilize the flexible backup option. However, unlike the case of increasing σ_1, the supplier does not resort to significant raise in the offered prices. Rather the supplier incentivizes the manufacturer by offering a lower options buy price. The differing acts of the supplier in the two cases can be attributed to the effect of σ_1 on x_1 in the former whereas there is no such case in the latter.

An increase in μ_2 shifts the period 2 demand to the right and hence, the manufacturer will require more supplies. This can be seen in the column for the base stock level in Table 6. The manufacturer does so by using both the supply sources i.e. the low cost main supplier and the flexible backup option.

Manufacturers Unit Costs and Game Behavior

The penalty cost on the lost sales, p directly affects the manufacturers base stock level, S_2. As p increases the base stock level also increases and vice versa. In such a case, the manufacturer books more supplies from both the sources (i.e. $Q + Q_M$). Please observe the change in the value of $S_2 - Q - Q_M$ with respect to p in Table 7. As $Q + Q_M$ represent the portion of the manufacturers requirement that it plans to fulfil from purchase from the two sources, an decrease in the value of $S_2 - Q - Q_M$ shows that the manufacturer books more quantities when the penalty, p increases and vice versa. Interestingly, as the penalty increases, the supplier also increases its options buy price and the manufacturer reacts by

TABLE 5. Effect of changing σ_2 (the co-efficient of variation changes)

σ_2	Q	Q_M	S_2	w_o	w_e	K
8.0	7.907	10.544	23.82	11.885	10.713	7.907
7.8	7.766	10.695	23.72	11.922	10.709	7.766
7.6	7.612	10.869	23.62	11.952	10.706	7.612
7.4	7.459	11.042	23.52	11.980	10.705	7.459
7.2	7.313	11.211	23.42	12.023	10.700	7.313
7.0	7.149	11.402	23.32	12.038	10.694	7.149
6.8	6.988	11.573	23.24	12.097	10.666	6.988
6.6	6.839	11.762	23.17	12.188	10.596	6.839
6.4	6.662	11.965	23.14	12.373	10.480	6.662
6.2	6.500	12.135	23.02	12.370	10.467	6.500
6.0	6.326	13.354	23.01	12.612	10.235	6.326

TABLE 6. Effect of changing μ_2 (the co-efficient of variation changes)

μ_2	Q	Q_M	S_2	w_o	w_e	K
25	7.378	16.154	28.05	11.620	11.237	7.378
24	7.338	15.200	27.15	11.803	11.052	7.338
23	7.309	14.231	26.21	11.900	10.923	7.309
22	7.259	13.281	25.26	11.991	10.813	7.259
21	7.209	12.337	24.29	12.008	10.756	7.209
20	7.149	11.402	23.32	12.038	10.694	7.149
19	7.079	10.482	22.41	12.176	10.508	7.049
18	6.983	9.600	21.53	12.339	10.290	6.983
17	6.887	8.714	20.67	12.545	10.013	6.887
16	6.675	7.861	19.87	12.860	9.628	6.675
15	6.637	7.050	19.07	13.124	9.264	6.637

shifting a portion of its backup option to low cost source. A similar but opposite conclusion is drawn for the case when the penalty of lost sales decreases.

TABLE 7. Effect of changing p

p	Q	Q_M	S_2	w_o	w_e	K
50	6.187	14.232	25.27	14.765	8.993	6.187
48	6.327	13.774	24.92	14.210	9.343	6.327
46	6.482	13.279	24.59	13.756	9.593	6.482
44	6.670	12.721	24.20	13.223	9.923	6.670
42	6.892	12.100	23.79	12.674	10.266	6.892
40	7.149	11.402	23.32	12.038	10.694	7.149
38	7.482	10.579	22.79	11.348	11.180	7.482
36	7.893	9.629	22.21	10.598	11.713	8.882
34	8.411	8.480	21.52	9.731	12.362	8.411
32	9.085	7.086	20.73	8.772	13.085	9.085
30	10.005	5.316	19.81	7.700	13.907	10.005

An obvious conclusion that can be observed in Table 8 is that when the purchase cost from the main supplier goes up the manufacturer reduces its order from the main supplier

and shifts them to the backup option. Infact, if we look at the value of $Q + Q_M$, it suggests that the manufacturers total purchase quantity decreases marginally as w_M increases. This forces the supplier to keep the options buy price unchanged but increases the exercise price. Increase in exercise price, however, reduces the base stock quantity.

TABLE 8. Effect of changing w_M

w_M	Q	Q_M	S_2	w_o	w_e	K
23.50	8.370	9.131	21.70	10.540	14.371	8.370
23.00	8.102	9.610	22.04	10.854	13.625	8.102
22.50	7.862	10.061	22.36	11.184	12.859	7.862
22.00	7.623	10.508	22.67	11.449	12.160	7.623
21.50	7.377	10.964	22.99	11.744	11.429	7.377
21.00	7.149	11.402	23.32	12.038	10.694	7.149
20.50	6.941	11.810	23.65	12.320	9.976	9.941
20.00	6.726	12.235	24.00	12.606	9.252	6.726
19.50	6.516	12.647	24.36	12.905	8.511	6.516
19.00	6.311	13.060	24.73	13.201	7.775	6.311
18.50	6.110	13.463	25.11	13.501	7.031	6.110

As shown in Table 9, change in the manufacturers inventory holding cost affects the basestock level and is inversely related to each other. As the holding cost increases manufacturers basestock level decreases and the manufacturer reduces its orders with the main supplier as well as the options buy quantity. The supplier, on the other hand, maintains its offered prices throughout suggesting that the manufacturers holding cost do not affect the suppliers pricing decision.

TABLE 9. Effect of changing h

h	Q	Q_M	S_2	w_o	w_e	K
5.50	6.870	11.243	22.71	12.311	10.429	6.870
5.00	6.926	11.275	22.81	12.226	10.518	6.926
4.50	6.975	11.306	22.93	12.170	10.574	6.975
4.00	7.023	11.348	23.05	12.120	10.621	7.023
3.50	7.095	11.366	23.19	12.082	10.654	7.095
3.00	7.149	11.402	23.32	12.038	10.694	7.149
2.50	7.210	11.431	23.46	12.001	10.733	7.210
2.00	7.282	11.451	23.62	11.978	10.754	7.282
1.50	7.352	11.479	23.77	11.954	10.778	7.352
1.00	7.422	11.509	23.95	11.954	10.778	7.422
0.50	7.492	11.539	24.13	11.954	10.778	7.492

Effect of Initial Inventory

To understand the results of Table 10 presenting the effects of the changes in the initial inventory on the equilibrium point we have to consider the relative distance between x_o and μ_1. As x_o decreases chances of x_1 being negative and hence, the possibility of the manufacture starting the period 2 with zero inventory increases (this may be restated as 'the certainty of starting period 2 with zero inventory increases). This reduction in uncertainty leads to marginal decrease in the manufacturers options buy quantity. Increase in x_o, however,

reduces the gap between x_o and μ_1 and hence, increases the uncertainty with x_1. This leads to the manufacturer utilizing more of the backup option and the supplier reacts to this by increasing the options buy price. But the higher price also reduces the manufacturers overall order quantity.

TABLE 10. Effect of changing x_0

x_0	Q	Q_M	S_2	w_o	w_e	K
10.50	7.307	10.244	23.61	12.569	10.075	7.307
10.00	7.268	10.553	23.57	12.508	10.154	7.268
9.50	7.245	10.806	23.52	12.428	10.260	7.245
9.00	7.201	11.044	23.49	12.378	10.330	7.201
8.50	7.177	11.234	23.39	12.189	10.543	7.177
8.00	7.149	11.402	23.32	12.038	10.694	7.149
7.50	7.129	11.532	23.24	11.856	10.886	7.129
7.00	7.105	11.646	23.13	11.626	11.125	7.105
6.50	7.095	11.716	23.01	11.364	11.397	7.095
6.00	7.086	11.784	22.91	11.164	11.611	7.086
5.50	7.079	11.822	22.71	10.704	12.071	7.079

7. CONCLUSIONS

Supply flexibility is very important for mitigating risks of supply disruption. Often firms involve backup supply sources when its main supplier is located in offshore location and is subjected to risks of disruption. In such a situation, use of a flexible contract ensures the backup supplier motivated and continuity of supply. We, in this paper, evaluated such a flexible contract in the form of options contract. We modeled the decision problem faced by a manufacturer who sources one of its important part from offshore locations and also engages a backup supplier. We modeled the problem as a Stackelberg game where the backup supplier is the leader and the manufacturer playing the role of a follower. Numerical experiments for some test problems gave important insights to the behaviour of the manufacturer and the supplier. We observed that uncertainty of any kind forces the manufacturer to utilize more of its backup options, however, the extent of such influence varies based on the nature of the uncertainty. The manufacturer gets affected most by the increase in the uncertainty of the supplies from the main supplier, followed by the uncertainty in the period 1 demand. In both the cases, the backup supplier exploits the situation by increasing its prices. The effect of increase in the uncertainty in period 2 demand, however, is marginal. In fact, the backup supplier even in some situations incentivizes the manufacturer by offering lower prices to encourage use of the backup options. Thus, we observe that the supplier displays opportunistic behaviour on some situations, whereas on other siatuations, incentivizes the manufacturer for buying more options. We see that the supplier incentivizes the manufacturer when the certainty of the on-hand inventory of the manufacturer before the later receives its supplies from the main supplier. Opportunistic behaviour is also displayed by the supplier when the the penalty cost of lost sales increases and incentivizes the manufacturer when penalty costs is lower.

There are many areas where the research presented in this paper can be extended. An immediate extension can be to carry out a comparative assessment of options contract with other flexible contracts like, quantity flexibility contract, capacity reservation contract and others. This paper assumes a forced compliance regime and therefore, a natural extension of this work will be to understand how the Stackelberg game between the manufacturer and

the supplier will be played under voluntary compliance regime. This research can also be extended to include various disruption scenarios like, a chance-based recovery of the main supplier, yield loss etc.

8. References

Atasoy, B., R. Gullu, T. Tan. (2012). Optimal inventory policies with non-stationary supply disruptions and advance supply information. Decision Support Systems, 53, 269-281.

Chen, K., T. Xiao. (2015). Outsourcing strategy and production disruption of supply chain with demand and capacity allocation uncertainties. International Journal of Production Economics, 170, 243-257.

Chen, K., Z. Xiaobo, Y. Zhou. (2012). A periodic-review inventory system with a capacitated backup supplier for mitigating suppy disruptions. European Journal of Operational Research, 219, 312-323.

Gallego, G., H. Hu. (2004). Optimal policies for production/inventory systems with finite capacity and Markov-modulated demand and supply processes. Annals of Operations Research 126, 2141.

Hou, J., A.Z. Zeng, L. Zhao. (2010). Coordination with a backup supplier through buyback contract under supply disruption. Transportation Research Part E: Logistics and Transportation Review, 46(6), 881-895.

Kouvelis, P., J. Li. (2008). Flexible Backup Supply and the Management of LeadTime Uncertainty. Production and Operations Management, 17(2), 184-199.

Latour, A. (2001). Trial by fire: a blaze in Albuquerque sets off major crisis for cell phone giants. Wall Streel Journal, January 29.

Li, Z., S.H. Xu, J. Hayya. (2004). A periodic-review inventory system with supply interruptions. Probability in the Engineering and Informational Sciences 18, 3353.

Minner, S. (2003). Multiple-supplier inventory models in supply chain management: A review. International Journal of Production Economics, 81, 265-279.

Nishiguchi, T., A. Beaudet. (1998). The Toyota Group and the aisin fire. Sloan Management Review, 40, 49-59.

Parlar, M., Y. Wang, Y. Gerchak. (1995). A periodic review inventory model with Markovian supply availability. International Journal of Production Economics, 42, 131136.

Snyder, L.V. (2014). A tight approximation for an EOQ model with supply disruptions. International Journal of Production Economics, 155, 91-108.

Sting, F.J., A. Huchzermeiyer. (2010). Ensuring responsive capacity: how to contract with backup suppliers. European Journal of Operational research, 207, 725-735.

Song, J.S., P. Zipkin. (1996). Inventory control with information about supply conditions. Management Science, 42, 14091419.

Zeng, A.Z., Y. Xia. (2015). Building a mutually beneficial partnership to ensure backup supply. Omega, 52, 77-91.

Zhu, S.X. (2015). Analysis of dual sourcing strategies under supply disruptions. International Journal of Production Economics, 170, 191-203.