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Incentive-Driven Two-Sided Matching and Spatial Capacity Allocation in Shared-Mobility Systems

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Abstract Motivated by free-float shared-mobility systems, we propose an integrated model of two-sided stochastic matching platforms, wherein agents on both sides respond to incentive instruments. From the platform's perspective, we formulate a strategic double-ended queueing model and operationalize it by jointly considering the optimal incentives as well as the spatial allocation of capacity. We identify an operational regime wherein the systems performance is competitive in almost all dimensions. We implement this optimization problem in a real case study, using a data set from a leading free-float bicycle-sharing system. Interestingly, we find that the hourly operational costs are lower in the rush hours, when the *market thickness* (transient service availability) is higher. Both stylized results and computational studies generate insights about fundamental trade-offs and triangular relationships among operational costs, capacity utilization rates and service levels.

Keywords Shared-mobility systems, double-sided queue, two-sided matching, capacity control, second-order conic programming.

1. Introduction

Starting from late 2016, free-float bicycle-sharing system such as Mobike or Ofo, is leading the next buzz in the shared-mobility industry. With an (estimated) combined 20 million new bikes to Chinese city streets in 2017 (2 million in 2016)[1], they quickly change the last-mile transportation preference of Chinese city-dwellers on a massive scale. However, this explosive growth ignores a negative externality as the free-float bicycles encroach public parking spaces, and responsive actions are taken by city governments to control the bicycle densities.[2]

In contrast to the current *repositioning* approach , we propose to use incentive instrument (price or reward adjustment) to balance the spatio-temporal supply and demand for free-float bicycles. Ambitious ride-sharing giants are entering this market to complement their product line.[3] Such incentive-driven mechanism has been long adopted and justified by

[1] https://www.ft.com/content/5efe95f6-0aeb-11e7-97d1-5e720a26771b.

[2] https://www.forbes.com/sites/bensin/2017/04/05/chinas-innovative-station-less-bike-shares-hit-obstacles-at-home-and-abroad/#73c5b1f84ff6.

[3] https://www.cnbc.com/2018/01/09/didi-chuxing-launches-bike-sharing-service.html

ride-sharing companies such as Uber or Lyft, and will be a logical next step for the bicycle-sharing systems. It has been confirmed that both customers and service providers respond to incentive instrument [4]. Therefore, unlike costly "repositioning", it is possible to achieve a better supply and demand coordination via incentive instrument.

Despite our focus on the free-float bicycle-sharing systems in this paper, the ideas and methodologies developed are applicable in wider range of shared-mobility systems. Compared to the well-understood ride-sharing systems, we identify the key new feature in the free-float bicycle-sharing systems as *market thickness*, defined loosely as the transient availability of service provider. In a ride-sharing system, there could be drivers in and out of the labor market and it suffice to consider drivers who are ready to serve. However, there are designated parking areas for idle bicycles. Our data suggest that the utilization rates for bicycles are low and a justification of such market thickness is necessary.

In this paper, we address the following research questions:

- How to characterize the market dynamics and systems performances of a free-float shared-mobility systems, when both the customers and the service providers respond to incentive instruments?
- How to design such systems in terms of optimal incentives and spatial capacity allocation to achieve high service level, low operational cost and high capacity utilization?

From the platform's perspective, we formulate an integrated optimization problem: At the strategic level, the platform decides the allocation of capacity. In the share-mobility application, this relates to the urban planning stage wherein the city governments decides the parking capacity allocation across the city (centralized big parking lots vs. decentralized smaller lots). At the operational level, optimal incentives are chosen to balance supply and demand. Overall, the platform maximizes long-run payoff subject to service level constraints. A wide range of auxiliary performance measures are calculated to evaluate different operating regimes.

This repositioning problem is well-received by the operations research literature, for example, pioneered by [13], and recent optimization development [15, 5, 7, 11]. There are also a few notable papers combining both strategic and operational level decisions. For example, [9] consider one-way car-sharing system (car2go), and model the market dynamics by standard queueing networks and formulate the fleet management problem as a mixed integer second-order cone program. [10] consider car-sharing systems (Zipcar), and solve the repositioning problem by branch-and-cut algorithms with mixed-integer, rounding-enhanced Benders cuts. [8] extend the fleet repositioning problem aiming to dynamically match the vehicle supply and travel demand under distributionally robust optimization reformulation. [14] consider a ride-sharing platforms by modeling the supply of drivers as servers in single-server queues. The market dynamics in [6] are more realistic by considering a multi-server queueing setup (limited but multiple drivers), and they compare the efficiency of the on-demand matching mechanism with the traditional street-hailing mechanism. [2] explore spatial price discrimination in the context of a ride-sharing platform that serves a network of locations. [1] considering closed-queueing networks wherein only vehicles are transient entities. However, the popular queueing networks typically overstep issues, for example, travel times among stations are modeled as service time. The doubled-sided queue in our paper is similar to that in [12].

The rest of this paper is organized as follows. Section 2 introduces our model setup. In Section 3, we carry out the analysis. In Section 4, we describe our computational method, and provide case studies. Section 5 concludes this paper.

2. The Basic Model

Market structure. Consider a city zoned into n different regions, across which rates for both usage (p_i) and delivers (r_i) differs. We consider a single-period setting, which can

be viewed as an arbitrary time-period during a day. We model the market structure by assuming that the riding demand for zone i is linear decreasing in its usage price, i.e.,

$$d_i = \lambda_i - \alpha_i p_i, \forall i \in I. \tag{1}$$

$\lambda_i > 0$ corresponds to market potential, i.e., baseline demand driven by the customers. $\alpha_i > 0$ (price elasticity) captures the fact that the service demand decreases as the price increases. Similarly, we assume that the supply of the service providers in zone i is linear decreasing in its usage rate, i.e.,

$$s_i = \mu_i + \beta_i r_i, \forall i \in I. \tag{2}$$

$\mu_i > 0$ corresponds to baseline arrival rates of the service providers to zone i without any reward. $\beta_i > 0$ (price elasticity) captures the fact that the service delivery increases as the reward increases. For stability consideration, we require that $s_i > d_i$.

The platform. To enable a tractable spatial pricing structure, we assume that transfers take place at demand and supply generation. The platform charges p_i for service in zone $i \in I$, and reward r_i for deliver service to that zone. In each zone, there is a fixed allocated capacity N_i (the buffer size for the service provider). We will define this expected number of available service provider as "market thickness", and since they are idling, there is a holding cost function h to penalize the market thickness. This conceptualization of market thickness has received emerging attention [3]. In the ride-sharing applications, the reward r_i represent the wage of drivers. This reflects a surge pricing in the region i. For bicycle-sharing, it reflects the discount to reach a less popular district. For example, the total transfer from an user to the platform for a trip starting from region i to region j is $p_i - r_j$. Theoretically, both p_i and r_i can be either positive or negative, which enables spatial price discrimination to incentivize a self-balanced redistribution. The capacity N_i captures the parking space for vehicles. For example, despite its convenience and universal availability, the free-float bicycle-sharing system also takes exorbitant on/off-street parking space, which is an encroachment of public interests. Therefore, under government intervention, the platform will limit the maximum number (N_i) of bicycles in different regions, due to limited parking areas.

Double-ended queues. We model both the numbers customers searching and the service providers available as two stochastic processes. To reduce the state-space (from a two-dimensional stochastic process to a single-dimensional one), we take the difference ($n_i(t)$) in the number of customers and the service providers in zone i at time t. Consequently, we are able to keep track of this quantity by a Markov process $\{n_i(t), t > 0\}$, driven by Poisson arrival of customers with rate d_i, and service providers with rate s_i. We assume that the matching process when both customers and service providers are available is instantaneous. The matched customers and service providers simply leave the system, with "service complete" at the instant of matching.

Hierarchy of decision making and sequence of events. We consider the matching platform's decision problem, which maximizes long-run revenue. The revenue is generated from a customer once he is matched to a service provider. Reward is paid to a service provider upon arrival to the corresponding region. In addition, a service level constraint will be considered to ensure that a customers' waiting time is not too long. Furthermore, market thickness will be penalized to control the density of service providers.

3. Analysis

Lemma 1. *The steady-state distribution of the difference in the number of customers searching for service provider and the number of service provider available is given by* $\pi_m = \lim_{t\to\infty} \Pr\{n_i(t) = x\}$ *in zone i:*

$$\pi_x = \left(\frac{s_i - d_i}{s_i}\right)\left(\frac{d_i}{s_i}\right)^{N_i+m}, \tag{3}$$

for $x = \{-N_i, -N_i+1, ..., 0, ..., \infty\}$. *The expected number of service providers (market thickness) in zone* i *is*

$$L_i = N_i - \frac{d_i}{s_i - d_i}\left[1 - \left(\frac{d_i}{s_i}\right)^{N_i}\right], \tag{4}$$

while the expected time for an arbitrary customer to find a service provider (service time) therein is

$$W_i^{'} = \frac{1}{s_i - d_i}\left(\frac{d_i}{s_i}\right)^{N_i}. \tag{5}$$

Lemma 2. *Assume that the system is stable* ($d_i < s_i$)*, the long-run steady-state rate of revenue generation is* $p_i \cdot d_i$.

The revenue generation rate is state-dependent since a payment transfer is measured at the moment when a successful matching take place. From an alternative perspective, this result is intuitive since there is no customer abandonment given long-run stability. Suppose the platform maximizes long-run payoff using the rate of revenue generation subtracted by the rate of subsidy cost. The platform's payoff can by calculated by

$$\pi = \sum_{i\in I} p_i \cdot d_i + r_i s_i \Pr\{n_i(t) \gtreqless -N_i\} = \sum_{i\in I} (p_i - r_i)(\lambda_i - \alpha_i p_i). \tag{6}$$

In this formulation, the stability constraint $\lambda_i - \alpha_i p_i < \mu_i + \beta_i r_i$ ensures the arrival rate of the service providers is always higher than the customers'. The system is stable because $\frac{d_i}{s_i} < 1$ and the quantities in **Lemma 1** are well-defined. In addition, a service level constraint will be considered to ensure that a customers' expected waiting time $W_i^{'} = \frac{1}{s_i - d_i}\left(\frac{d_i}{s_i}\right)^{N_i} \le T_i$. The waiting time reflects a cost to search an available bicycle in the entire region. Therefore, the platform's decision problem is formulated as follows:

$$\max_{(p_i, r_i, N_i)_{i\in I}} \pi = \sum_{i\in I} \pi_i = \sum_{i\in I} (p_i - r_i)(\lambda_i - \alpha_i p_i),$$

subject to

$$\begin{aligned} \lambda_i - \alpha_i p_i &< \mu_i + \beta_i r_i, \forall i \in I, \\ \frac{\left(\frac{\lambda_i - \alpha_i p_i}{\mu_i + \beta_i r_i}\right)^{N_i}}{\mu_i + \beta_i r_i - \lambda_i + \alpha_i p_i} &\le T_i, \forall i \in I, \\ \sum_{i\in I} N_i &\le N. \end{aligned} \tag{P-1}$$

In the basic model, we will be maximizing the platform's payoff, subject to the service level constraint. The market thickness serves as a secondary performance measure. Direct analysis along this direction has been done in similar models [12]. Unfortunately, this problem admits little analytical results due to the highly nonlinear service level constraint $W_i^{'} < T_i$. In what follows, we consider a heavy traffic regime such that $s_i = d_i + \delta\sqrt{d_i}$ for a small $\delta > 0$, know as the *square-root staffing rule*. The coefficient δ captures the extra supply needed to satisfy demand, which measures the service level, i.e., a higher δ leads to better service (shorter customer waiting time).

Proposition 1. *Assume that* $\delta < \frac{1}{T_i N_i^2 e}$ *(*e *being the Euler's number) and* N_i *being large. The service level constraint* $W_i^{'} \le T_i$ *can be approximated by* $W_i^{'} \approx \frac{1}{\delta\sqrt{d_i} + N_i\delta^2} \le T_i$.

Therefore, the market thickness is given by

$$L_i = N_i - \frac{d_i}{s_i - d_i}\left[1 - \left(\frac{d_i}{s_i}\right)^{N_i}\right] \approx N_i - \frac{1}{\delta}\sqrt{d_i} + \frac{d_i}{\delta\sqrt{d_i} + N_i\delta^2}. \tag{7}$$

Proposition 2. *The following statements are true under the heavy traffic regime.*

- *The revenue-maximizing incentives are given as follows:*

$$\begin{aligned} p_i &= \frac{\lambda_i}{\alpha_i} - \frac{1}{\alpha_i\delta^2 T_i^2} + \frac{2N_i}{\alpha_i T_i} + \frac{\delta^2 N_i^2}{\alpha_i}, \\ r_i &= \frac{1}{\beta_i\delta^2 T_i^2} - \frac{2N_i - 1}{\beta_i T_i} - \frac{\mu_i}{\beta_i}. \end{aligned} \tag{8}$$

  The corresponding payoff is

$$\pi = \sum_{i\in I} \frac{\left[(\lambda_i\beta_i + \mu_i\alpha_i)\delta^2 T_i^2 - (\alpha_i + \beta_i)\left(1 - 2\delta^2 N_i T_i\right)\right]\left(1 - 2\delta^2 N_i T_i\right)}{\alpha_i\beta_i\delta^4 T_i^4}. \tag{9}$$

- *The optimal spatial capacity allocation is given by*

$$N_i = \frac{1}{2\delta^2 T_i} - \frac{(\lambda_i\beta_i + \mu_i\alpha_i)T_i}{4\alpha_i\beta_i} + \frac{\alpha_i\beta_i T_i^2 \max\{\gamma, 0\}}{8(\alpha_i + \beta_i)}, \tag{10}$$

$$\gamma = \frac{N - \sum_{i\in I}\left[\frac{1}{2\delta^2 T_i} - \frac{(\lambda_i\beta_i + \mu_i\alpha_i)T_i}{4\alpha_i\beta_i}\right]}{\sum_{i\in I}\frac{\alpha_i\beta_i T_i^2}{8(\alpha_i + \beta_i)}}. \tag{11}$$

- *The market dynamics in the difference between the customers and the service providers $\{n_i(t), t > 0\}$ can be approximated by the Reflected-Brownian Motions (RBM) with a negative drift $d_i - s_i$ and a standard deviation $\sqrt{2d_i}$, starting from $t = 0$, i.e., $\{n_i(t) + N_i, t > 0\} =_{s.t.} RBM_{n_i(0)+N_i}(d_i - s_i, 2d)$, with the following stationary distribution:*

$$\lim_{t\to\infty} \Pr\{n_i(t) > x\} \approx \exp\left\{-\frac{\delta^2 T_i (x + N_i)}{1 - 2\delta^2 T_i N_i}\right\}, \forall x \geq -N_i. \tag{12}$$

With the RBM characterization of the market dynamics, we can also directly evaluate the systems performance based on RBM. The consistency between these two approaches will be ensured by the interchangeability of the time limit as $t \to \infty$ and the diffusion limit. For example, the expected queue length can be calculated by $\lim_{t\to\infty} \frac{n_i(t)}{t} = \frac{1}{\delta^2 T_i} - 3N_i$, from the stationary distribution under RBM. This is consistent with the original Birth-Death Process, i.e.,

$$\lim_{t\to\infty} \frac{n_i(t)}{t} = \frac{d_i}{s_i - d_i} - N_i = \frac{1}{\delta^2 T_i} - 3N_i. \tag{13}$$

Our approximation paradigm (first calculate performance measures, and then take heavy traffic limits) can be easily operationalized as will be demonstrated in the computational studies. The understanding of the market dynamics also generates additional operational insights. For example, we know that $\sup_{0\leq t<T} |z(t)| = O(\log T)$, almost surely, for a RBM with a negative drift $\{z(t)\}$. Therefore, $\sup_{0\leq t<T} n_i(t) = O(\log T) - N_i$, which can be useful to estimate the service level during a rush hour: suppose that the rush hour length being $T \to \infty$, then $N_i \to O(\log T)$ is sufficient to maintain a worst-case service availability.

In the heavy traffic regime, the service level constraints are binding and the customers' expected waiting times are (T_i). The *market thickness* L_i is defined as the expected number of idle service providers. The *capacity utilization rate is* defined as $\frac{N_i - L_i}{N_i}$. The utilization rate turns out to be

$$1-\delta^2 N_i T_i \approx \frac{1}{2}+\frac{\delta^2(\lambda_i\beta_i+\mu_i\alpha_i)T_i^2}{4(\beta_i+\alpha_i)}, \tag{14}$$

which is approximately 50% as $\delta \to 0$.

When δ is small, the number of the waiting customers is

$$\frac{1}{\delta^2 T_i}+1-2N_i+\delta^2 N_i^2 T_i \approx \frac{1}{4\delta^2 T_i}+1, \tag{15}$$

which will be a long queue. In addition, in terms of the success rate of a random service provider being matched with a customer (also the time-average fraction), we define the *server's matching rate* as $\Pr\{n_i(t) \gtrless -N_i\} = \frac{d_i}{s_i}$. Heavy traffic approximations achieve a 100% matching rate.

4. Computations and Case Studies

In this section, we shall demonstrate the possibility to operationalize the stylized model. We formulate a basic revenue maximization problem, penalized by the market thickness, and approximate the service level constraints with the appropriate accuracy. The optimization problem is formulated as follows.

$$\max_{(p_i,r_i,N_i)_{i\in I}} \pi = \sum_{i\in I} p_i\left(\lambda_i-\alpha_i p_i\right) - r_i(\mu_i+\beta_i r_i) - h\left(N_i - \frac{1}{\delta}\sqrt{\lambda_i-\alpha_i p_i} + \frac{\lambda_i-\alpha_i p_i}{\delta\sqrt{\lambda_i-\alpha_i p_i}+N_i\delta^2}\right)$$

subject to

$$\begin{aligned} \lambda_i-\alpha_i p_i+\delta\sqrt{\lambda_i-\alpha_i p_i} &\le \mu_i+\beta_i r_i, \forall i\in I,\\ \frac{1}{\delta\sqrt{\lambda_i-\alpha_i p_i}+N_i\delta^2} &\le T_i, \forall i\in I,\\ \sum_{i\in I} N_i \le N, N_i &\in \mathbf{Z}^+, \forall i\in I. \end{aligned} \tag{P-4}$$

Conceptually, this problem can be extended in multiple directions by adding more realistic operational constraints. To highlight the core trade-off among profitability, utilization and service level, we discuss the computational aspects for this basic program and complement theoretic results with a real case study. To reformulate this problem, we take $\sqrt{\lambda_i-\alpha_i p_i} \triangleq x_i$ as a variable and solve the following equivalent cost-minimization problem.

Proposition 3. *The reformulation can be solved by the following mixed-integer Second-Order Conic Programming (SOCP) problem*

$$\min_{(r_i,x_i,w_i,t_i,z_i,N_i)_{i\in I}} \sum_{i\in I} w_i + t_i - hN_i - \frac{h}{\delta}x_i + hz_i \tag{16}$$

subject to

$$\left\| \begin{pmatrix} x_i \\ \frac{y_i-1}{2} \end{pmatrix} \right\|_2 \le \frac{y_i+1}{2}, \forall i\in I, \tag{17}$$

$$\left\| \begin{pmatrix} y_i-\frac{\lambda_i}{2} \\ \frac{\alpha_i w_i+\left(\frac{\lambda_i}{2}\right)^2-1}{2} \end{pmatrix} \right\|_2 \le \frac{\alpha_i w_i+\left(\frac{\lambda_i}{2}\right)^2+1}{2}, \forall i\in I, \tag{18}$$

$$\left\| \begin{pmatrix} r_i+\frac{\mu_i}{2\beta_i} \\ \frac{\frac{t_i}{\beta_i}+\left(\frac{\mu_i}{2\beta_i}\right)^2-1}{2} \end{pmatrix} \right\|_2 \le \frac{\frac{t_i}{\beta_i}+\left(\frac{\mu_i}{2\beta_i}\right)^2+1}{2}, \forall i\in I, \tag{19}$$

$$\left\| \begin{pmatrix} x_i \\ \frac{\delta x_i + \delta^2 N_i - z_i}{2} \end{pmatrix} \right\|_2 \leq \frac{\delta x_i + \delta^2 N_i + z_i}{2}, \forall i \in I, \tag{20}$$

$$\left\| \begin{pmatrix} x_i + \frac{\delta}{2} \\ \frac{\beta_i r_i + \mu_i + \frac{\delta^2}{4} - 1}{2} \end{pmatrix} \right\|_2 \leq \frac{\beta_i r_i + \mu_i + \frac{\delta^2}{4} + 1}{2}, \forall i \in I, \tag{21}$$

$$\delta x_i + \delta^2 N_i \geq \frac{1}{T_i}, \forall i \in I, \tag{22}$$

$$\sum_{i \in I} N_i \leq N, x_i, r_i, w_i, t_i, z_i \in \mathbb{R}^+, N_i \in \mathbf{Z}^+.$$

Our data set is from a leading free-float bicycle-sharing company, which records over 3 million trips in a major Chinese city, consisting of unique trip ID, bicycle ID, starting and ending time as well as GPS locations. We begin by dividing the city into 100 regions, and calculate the demand and supply rates in every region. Under a granularity of 100 regions, the proportion of cross-region trips is low. This in turn explains why we focus on the market dynamics *within* regions, rather than the networked traffic flows *across* regions. We choose parameter combinations such that the market thickness penalty is much smaller than the scale of incentives.

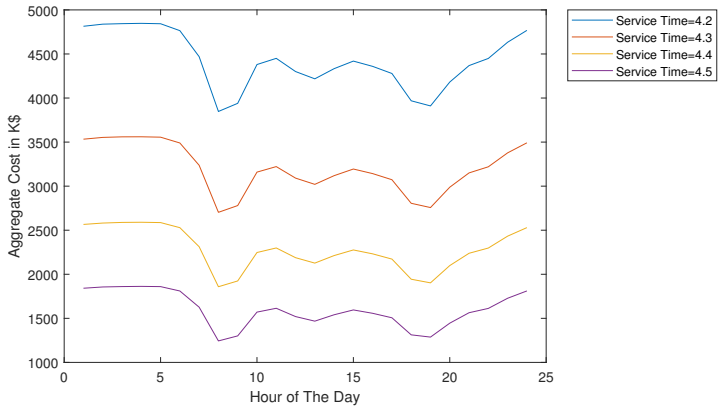


FIGURE 1. Hourly operating costs under different service time guarantee.

In Figure 1, we plot the aggregate costs over 24 hours in a day, under different service time guarantees. The operational costs are calculated hourly. We find that the operational cost decreases in the service time guarantee, as it is easier to achieve our target under a lower service level. Interestingly, the hourly operational costs are lowest in the rush hours. This is because under our randomly generated incentive sensitivities, the platform is essentially subsidizing the customers to redistribute the bicycles to balance the real-time supply and demand. The aggregate subsidy costs are lowest during the rush hours when the market is thick and easy to balance.

5. Conclusion

In this paper, we propose an integrated model in shared-mobility systems based on a two-sided matching platform. We model the stochastic processes of the numbers of both service providers and customers using double-ended queues, wherein agents on both sides respond to incentive instruments. In particular, we examine the market dynamics, and using heavy

traffic approximations to characterize the systems performance. From the platform's perspective, we formulate an optimization problem that jointly determines the the spatial capacity allocations as well as the optimal incentives. A wide range of auxiliary performance measures are calculated to evaluate different operational regimes. We implement this optimization problem in a real case study, which generates insights concerning the triangular relationship between operational costs, capacity utilization rates and service levels. Methodology-wise, this analytical paradigm can be applied to other applications related to the resource allocation in matching markets, sharing economies and two-sided platforms.

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